Lecture I

The Sonar Equation and Signal Detection

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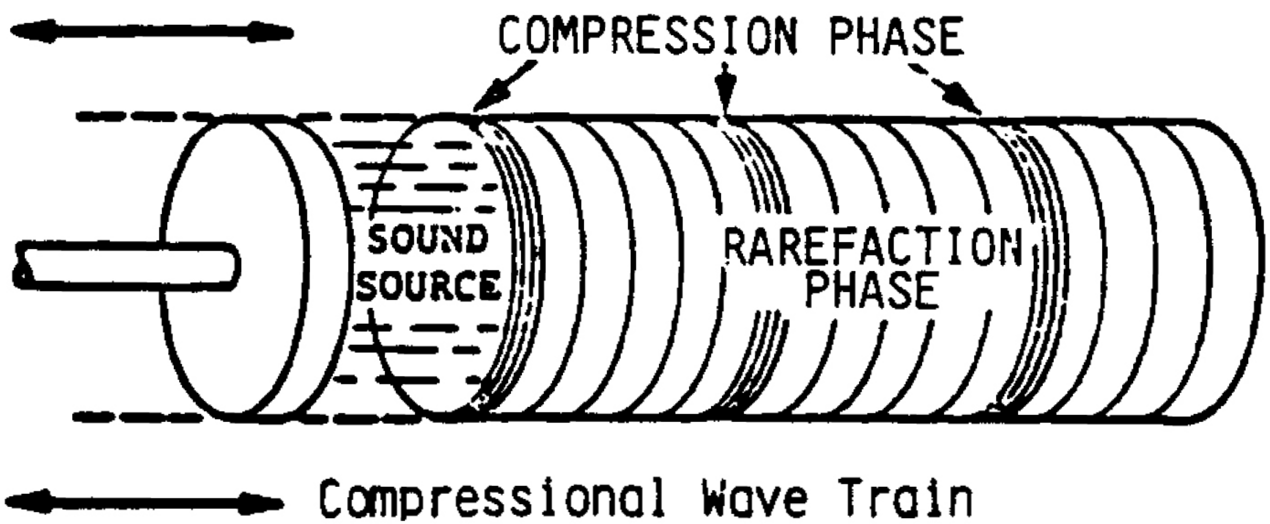
What is Sound?

Sound is a mechanical wave motion propagating in an elastic medium. Associated with this wave motion are changes in the local pressure and density.

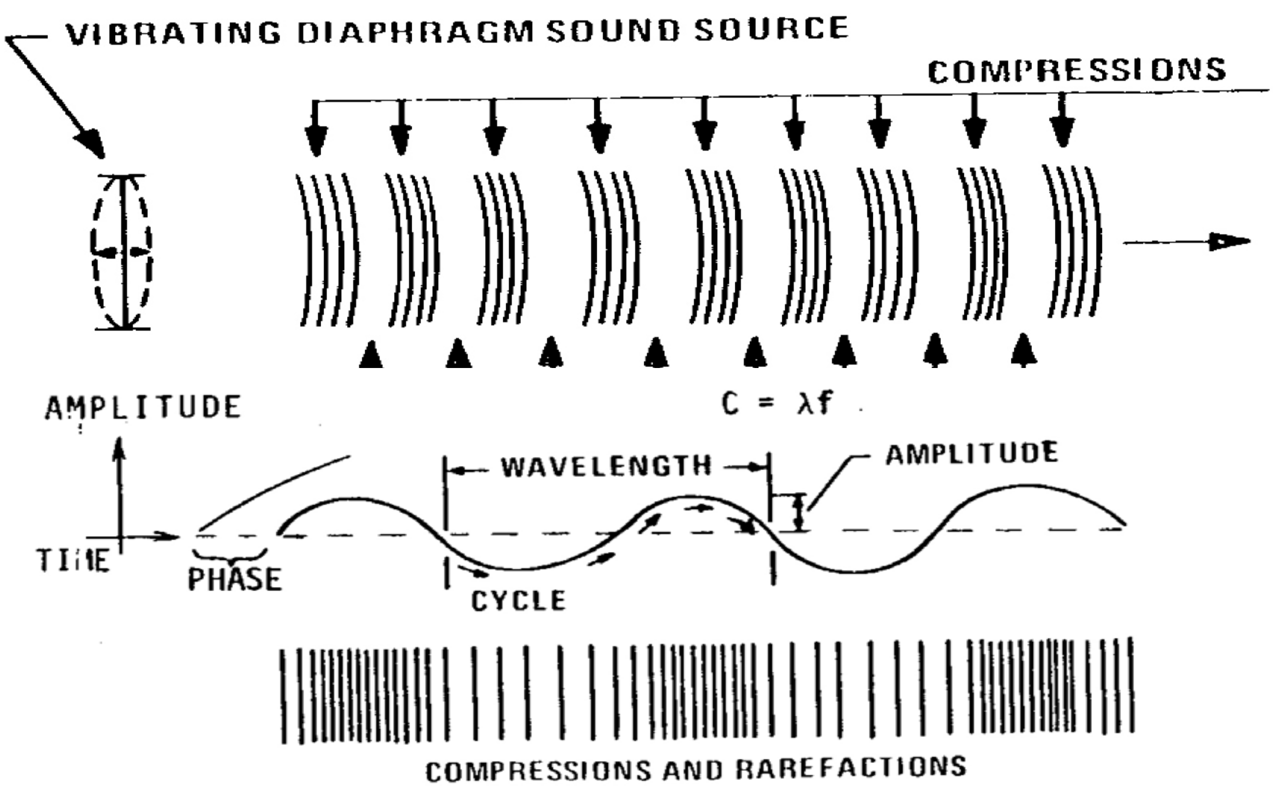
Sonar, an acronym for <u>SO</u>und <u>NA</u>vigation and <u>R</u>anging, uses sound energy (generally underwater) to transmit information.



Longitudinal Waves

COMPRESSIONS AND RAREFACTIONS OF LONGITUDINAL WAVES

λ = WAVELENGTH
f = FREQUENCY
c = SPEED OF SOUND



Terminology

Psycho/Physiological — Physical

Pitch ⟷ Frequency

(Nominal) **Audible Range: 20 –20,000 Hz**

Speech Range : 100 – 4,000 Hz

Infrasonic : Below audible range

Ultrasonic : Above audible range

Loudness ⟷ Intensity

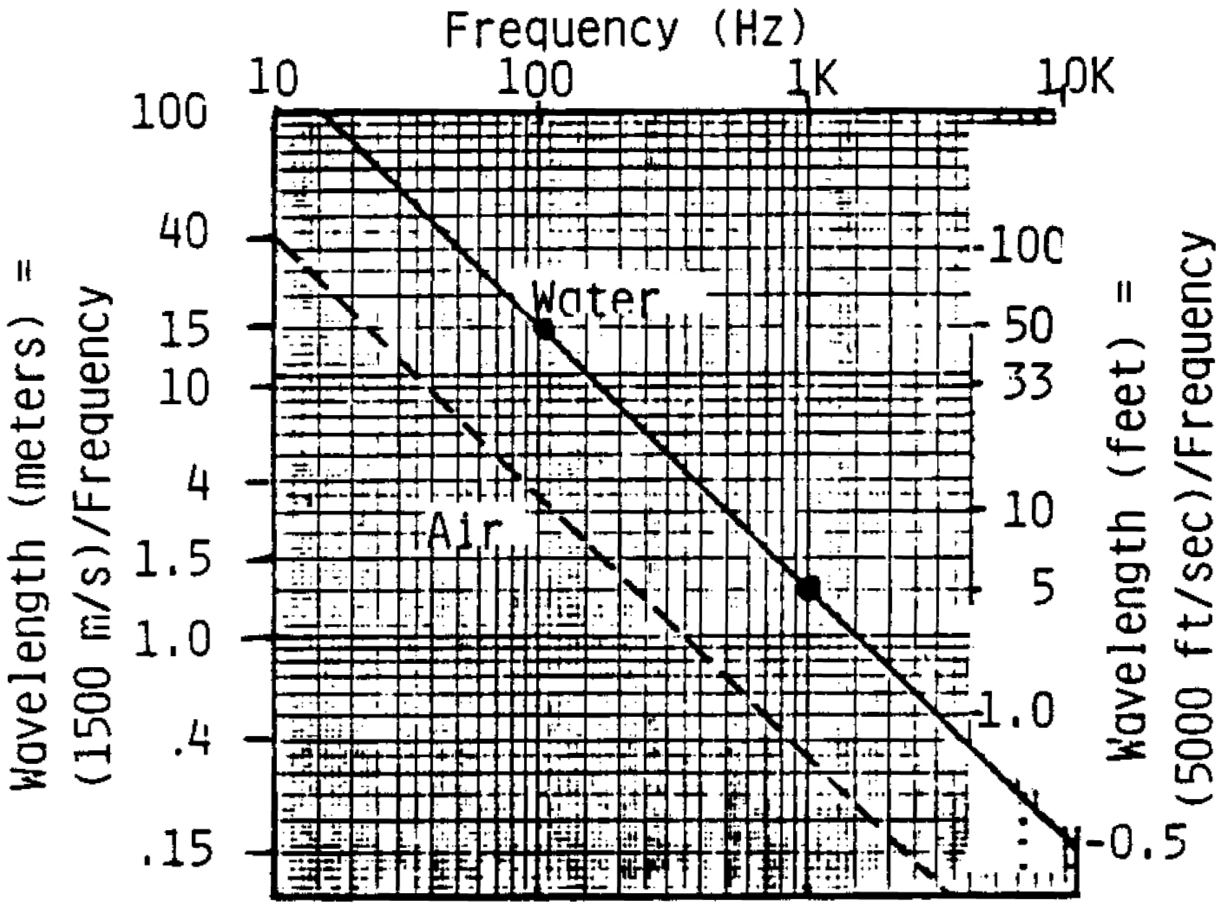
Intensity = 10 $LOG_{10}\, I_1/I_0$ = 20 $LOG_{10}\, P_1/P_0$ Decibels (dB)

Quality ⟷ Waveform Spectrum, Relative amplitudes/phases

Nominal Speed of Sound

Material	Speed of Sound (m/s)	Density (g/cm^{3})
Aluminum (rolled)	6420	2.7
Stainless Steel	5790	7.9
Rubber, Gum	1550	0.95
Fresh Water (25^{o}C)	1498	0.998
Sea Water (25^{o}C)	1531	1.025
Air	331	0.0012
Sea Water	1430$\rightarrow$1530	

Acoustic Wavelength versus Frequency

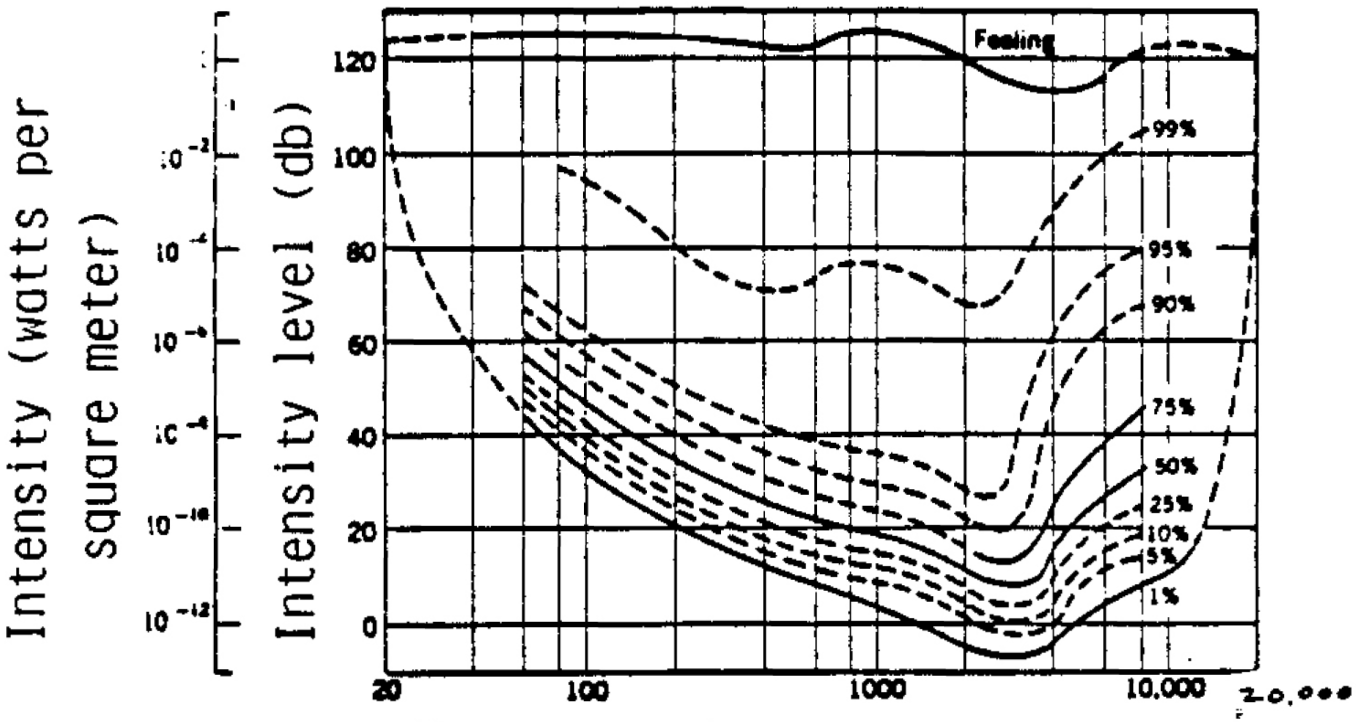


Wavelength = sound speed/ frequency

$$\lambda = c/f$$

Sensitivity of the Human Ear

0 dB = 0.0002 dyne/ sq. cm



Percentage of individuals who can hear of various intensities.

Shortly & Williams (53)

Doppler Shift (Passive Sonar)

When a sound source is moving with respect to an acoustic observer, the frequency of the sound is shifted.

$$f_0 = f_s \frac{c + V_0}{c - V_s}$$

f_0 = observed frequency

f_s = frequency at the source

c = speed of sound

V_0 = speed of observer approaching sound source = $V_1 \cos \Theta$

V_s = speed of source approaching observer = $V_2 \cos \Theta$

Θ = bearing angle

Doppler shift is about 0.35 Hz per knot, about 1% at 30 knots

Doppler Shift (Active Sonar)

In active sonar, the approaching target received a signal which is shifted higher in frequency (up-Doppler), then reradiates as a moving source approaching the receiver, causing a doubling of the Doppler shift.

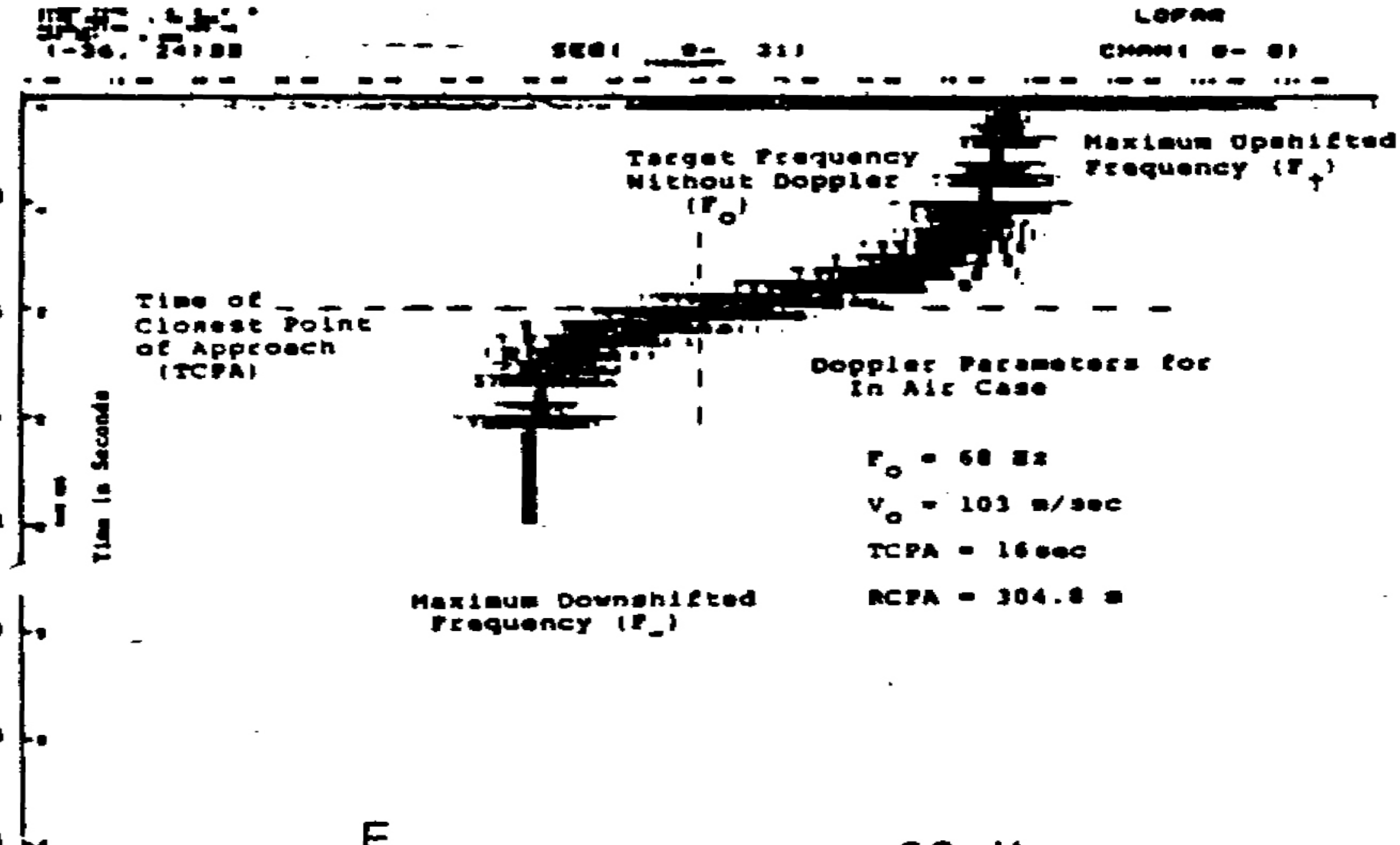
$$\frac{\Delta f}{f} \approx \frac{2\,(V_O + V_S)}{C - (V_O + V_S)} = 6.9 \times 10^{-4}$$

$\triangle$f = 0.69 Hz/(knot) (KHz) [ACTIVE]

REMEMBER

Approaching target produces an up-Doppler.
Receding target produced a down-Doppler.
Active sonar Doppler shift is double that of passive sonar
(about 2% at 30 knots)

Lofargram of Simulated Aircraft Overflight



$$F(t) = \frac{F_{source}}{1 - \frac{V_o}{C_o}\cos\theta} = \frac{68\ \text{Hz}}{1 - \frac{103}{331}\cos\theta}$$

Decibel (dB)

Decibels are 10 times the logarithm (to base 10) of the ration of a power or power-related quantity divided by a reference level.

I (dB re 1 watt/cm2) = 10 log I (watts/cm2)

Iref

(watt/cm2)

Remember the following properties of logarithms:

$$\log 10^{X} = X$$

$$\log [10^{X}\ 10^{Y}] = \log 10^{(X+Y)} = X+Y$$

To convert from dB to linear units

$$i \text{ (linear ratio)} = 10^{(0.1)I}$$

Where i = linear ration normalized by reference unit

I = dB value.

Intensity/Pressure/Acoustic Impedance

Intensity is the (sound) power flow per unit area

$$I = P^2 / \quad \rho c$$

where P = pressure amplitude

ρc = acoustic impedance

ρ = density

c = speed of sound

$$\text{Intensity} = 10 \log I/I_{ref}$$

$$= 10 \log \quad (P/P_{ref})^2$$

$$= 20 \log \quad P/P^2$$

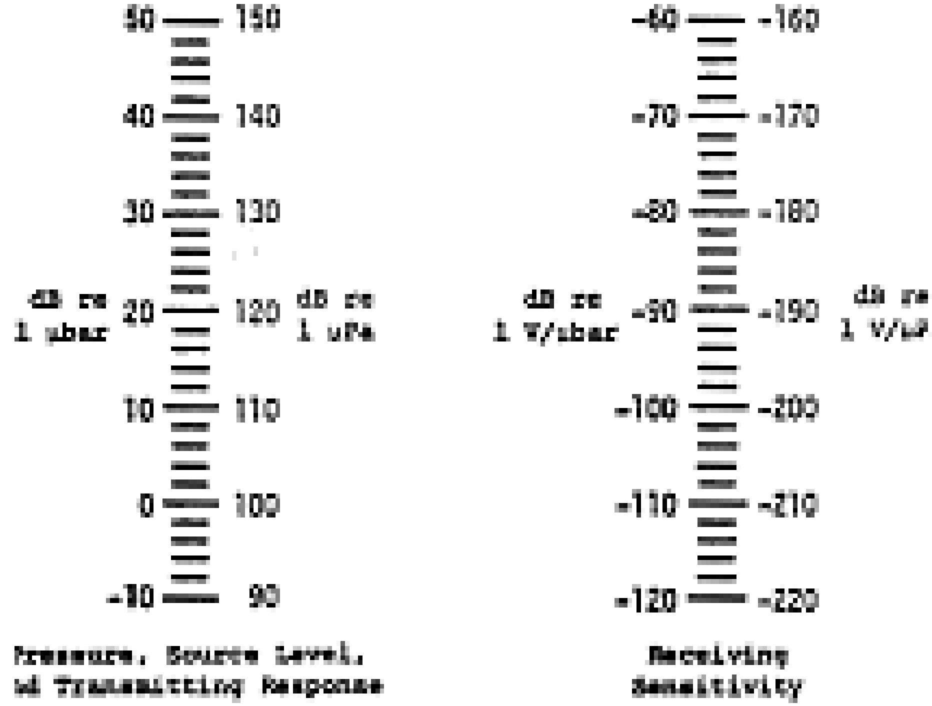
Pressure Units

- Force per unit area

- MKS units:
  1 Pascal = 1 Newton/m^2

- CGS units:
  1 μbar = 1 dyne/cm^2

-New reference unit
  1 μPascal = 10^{-6} Newton/m^2

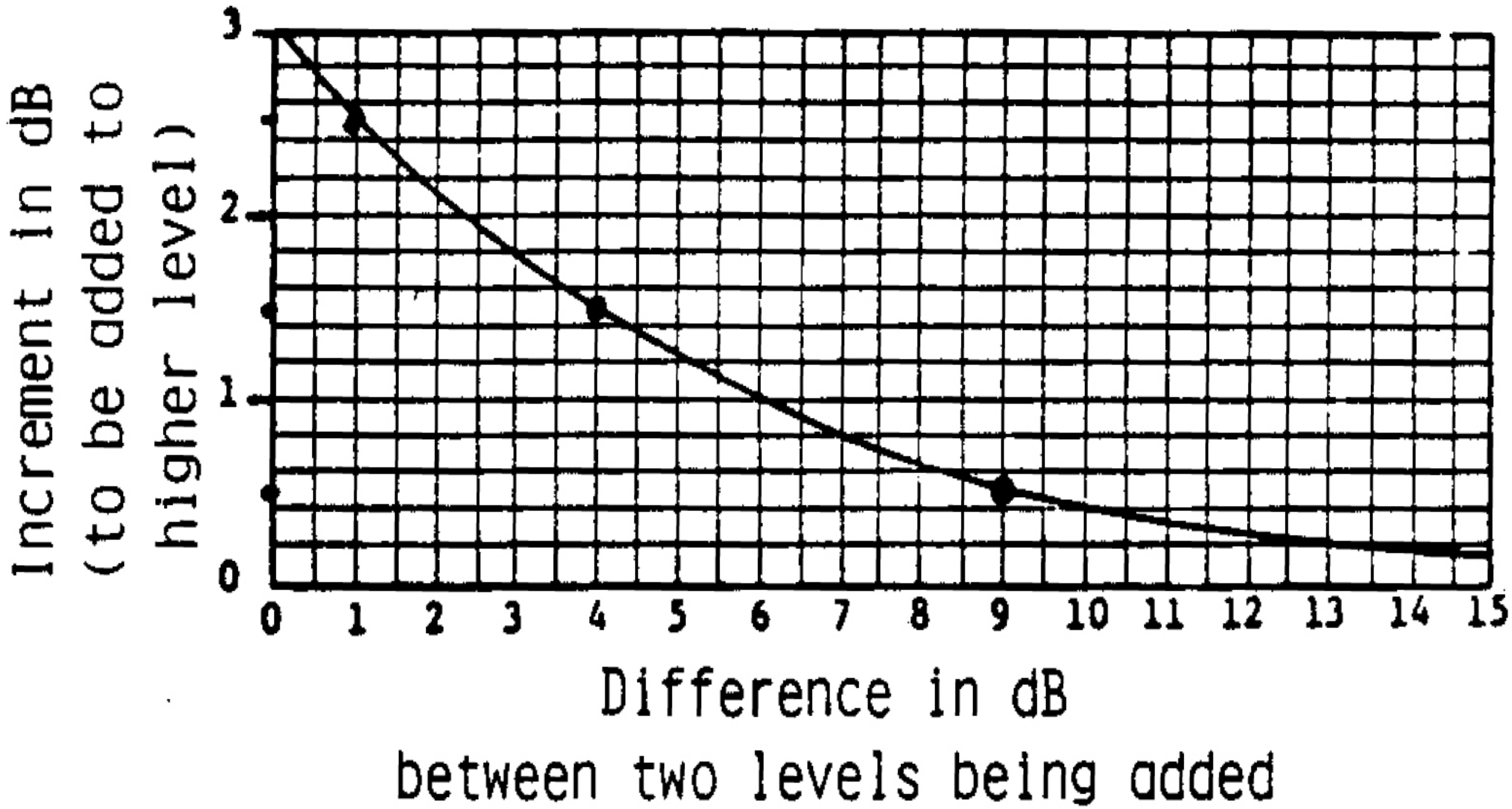


dB re 1 μPa = dB re 1 μbar + 100 dB

dB re 1 μPa = dB re .0002 μbar + 26 dB

Power Addition

It is sometimes necessary to sum powers of two numbers which are expressed by dB. The numbers cannot be simply summed in dB form, as that would correspond to multiplication. In order to sum power, first convert from dB to the original units, add, and then express the result in dB. Alternately, the following curve can be used.



EXAMPLE FOR THE COMBINATION OF SHIPPING NOISE AND WIND WAVE NOISE

Light Shipping (100 Hz) = 60 dB re 1 μPa

Wind-Wave (100 Hz, SS3) = 63 dB re 1 μPa

Total Noise = 63 + 1.8 dB re 1 μPa

EXAMPLE FOR DETECTION THRESHOLD = -5 dB

$(S/N)_{REQ}$ = -5 dB

$[(S+N/N)]_{REQ}$ = 1.2 dB

More Examples

If the acoustic pressure in a plane wave is increased by a factor of 3, what is the change in level of the sound?
If the acoustic intensity in a plane wave is increased by a factor of 3, what is the change in level of the sound?
If the acoustic pressure in a plane wave is reduced to 1/4, what is the change in level of the sound?
If the acoustic intensity in a plane wave is reduced to 1/10, what is the change in level of the sound?

Yet More Examples

At a certain location the noise level due to oceanic turbulence is 70 dB, the noise level due to distant shipping is 73 dB, and the self-noise level is 73 dB. What is the total noise level?

Bathymetry

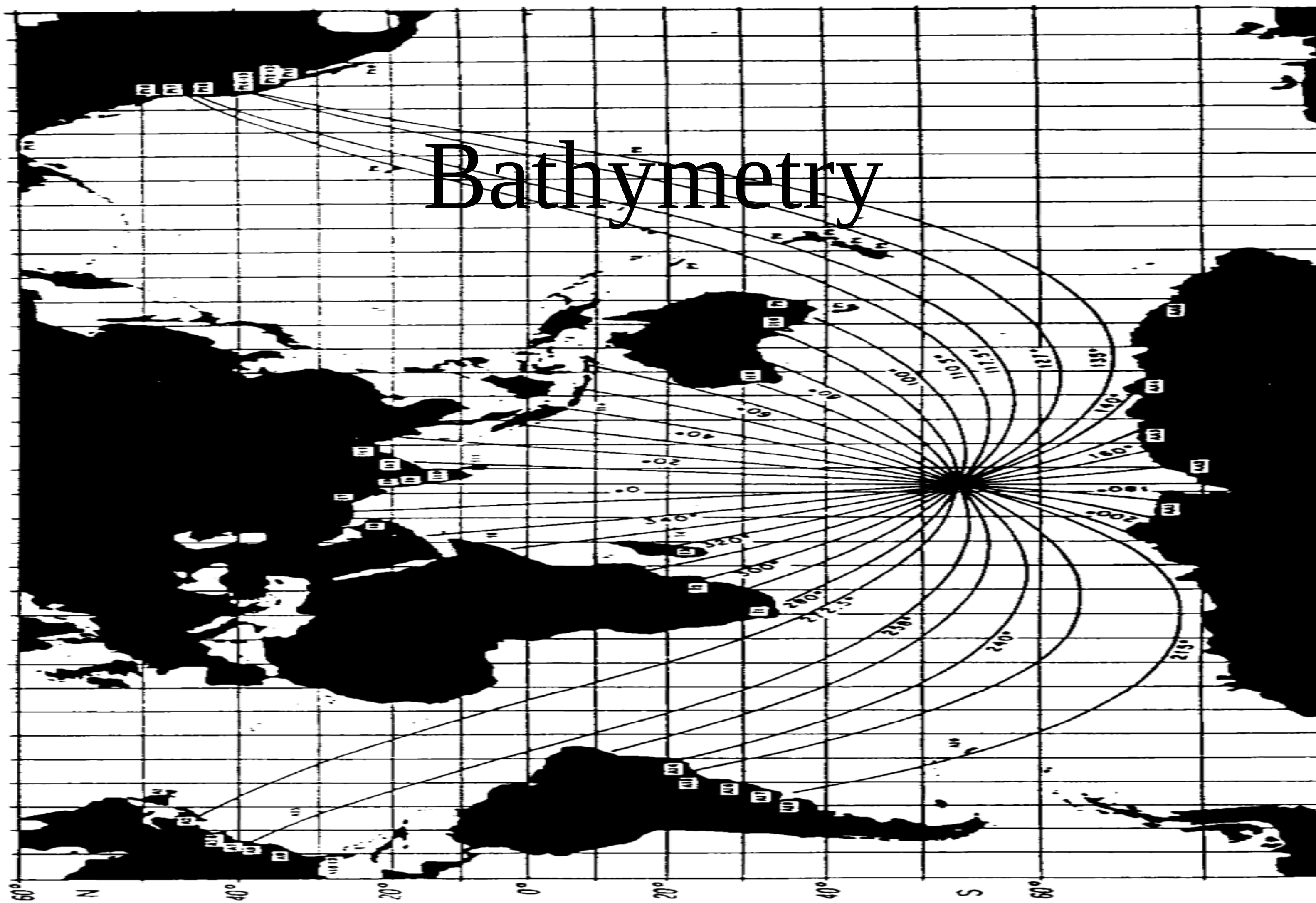
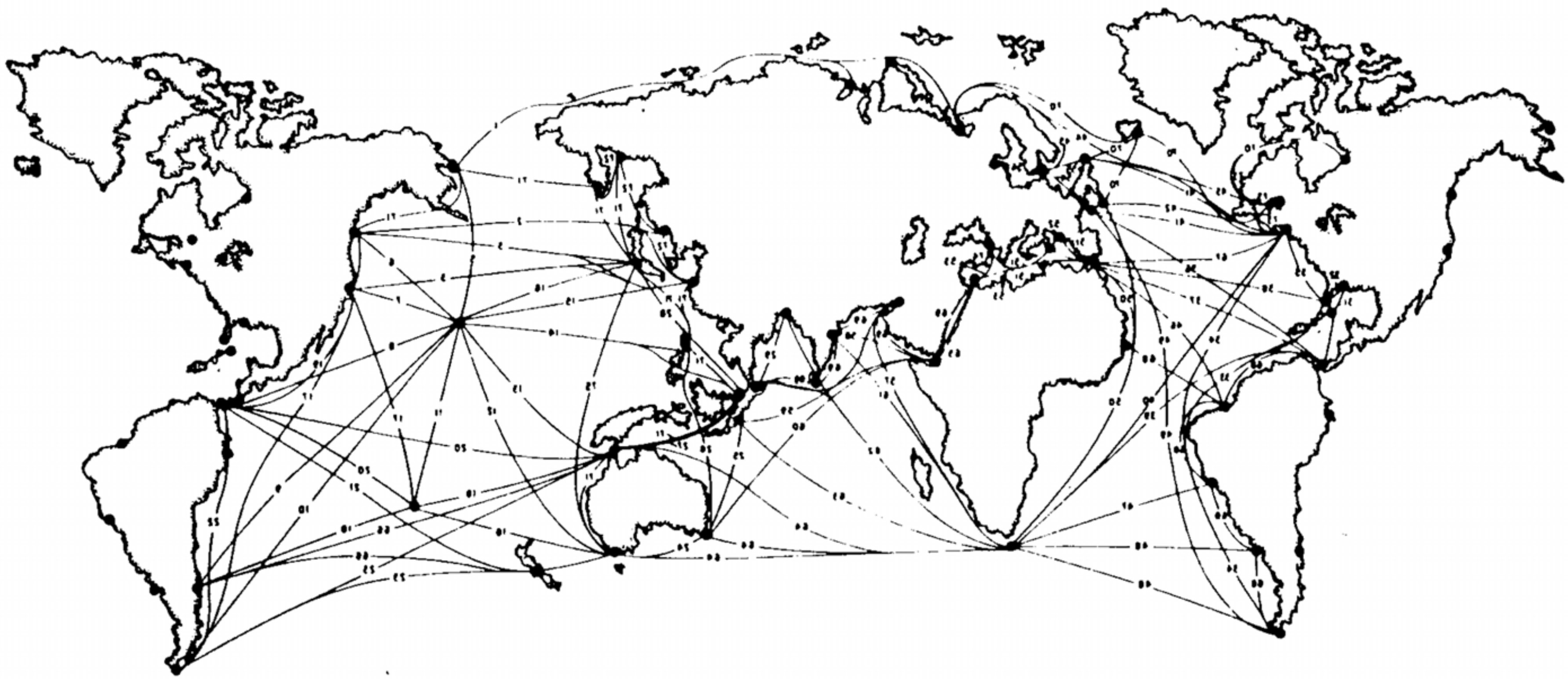


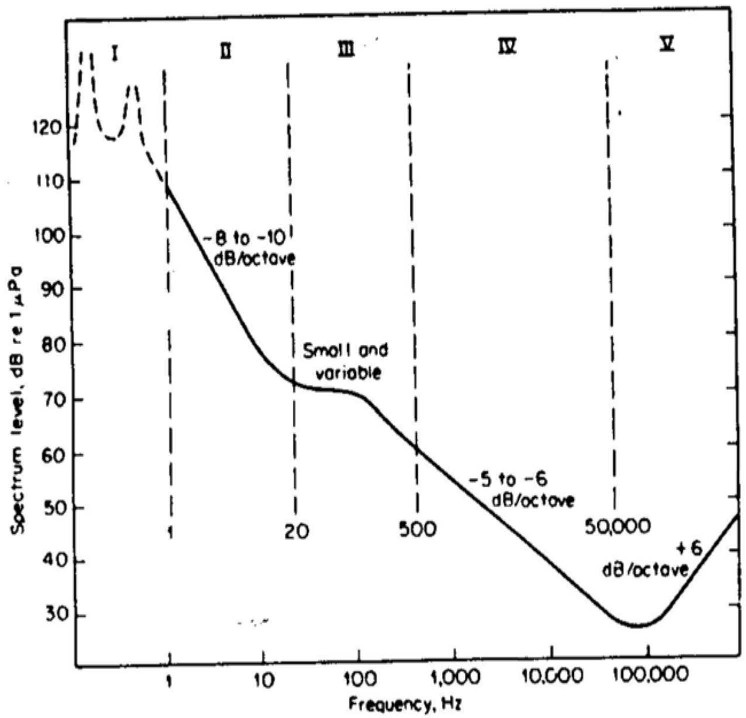
Illustration of bathymetrically unimpeded ray paths emanating from Herd Islands with the indicated launch angles. All rays are refracted geodesics along the surface containing the sound channel axis. White boxes indicate locations of oceanographic research stations capable of receiving acoustic signals. (Munk and Forbes, 1989; J. Phys. Oceanography, No. 19, 1965-78; copyright by the American Meteorological Society)

Generalized Shipping Routes

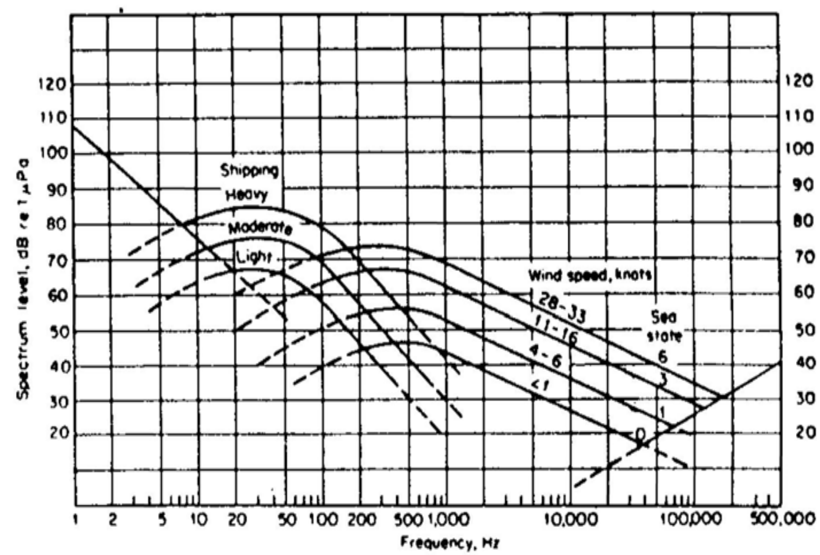


Generalized shipping routes (Solomon et al., 1977).

Noise Spectra



General spectrum of deep-sea noise showing five frequency bands of differing spectral slopes; the slopes are given in decibels per octave of frequency (Urick, 1983; *Principles of Underwater Sound*, 3rd edn;



· Average deep-sea ambient noise spectra (Urick, 1983; *Principles of Underwater Sound*, 3rd edn;

Bathymetry and Sound Speed Structure

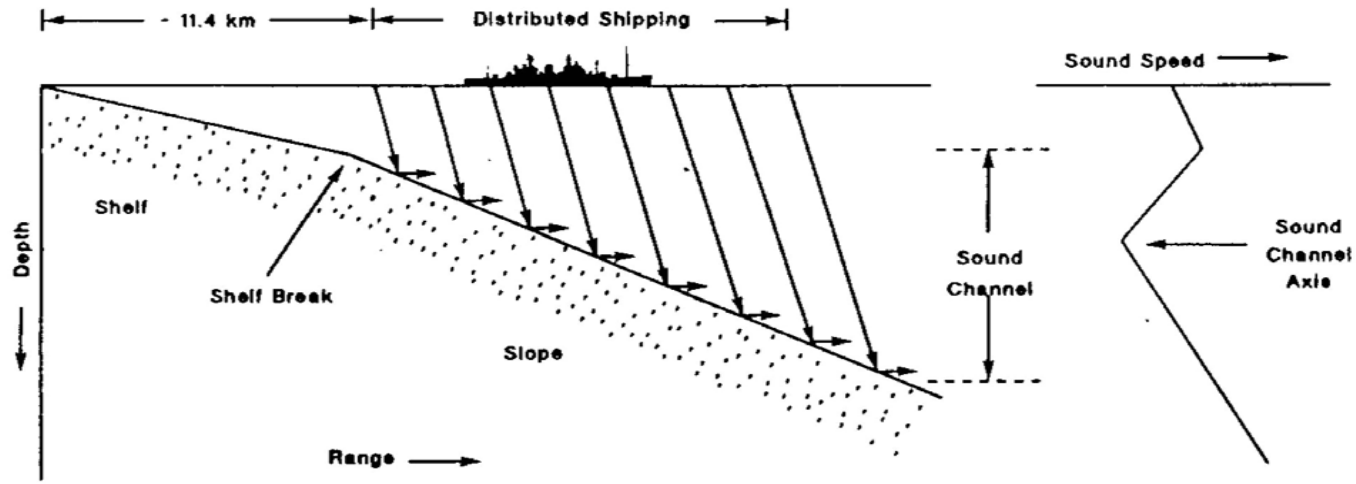
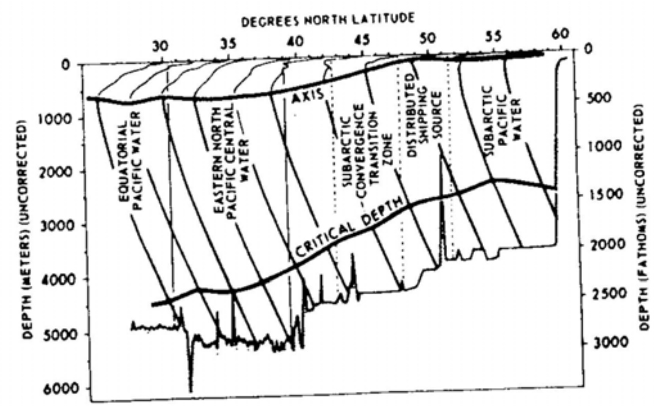
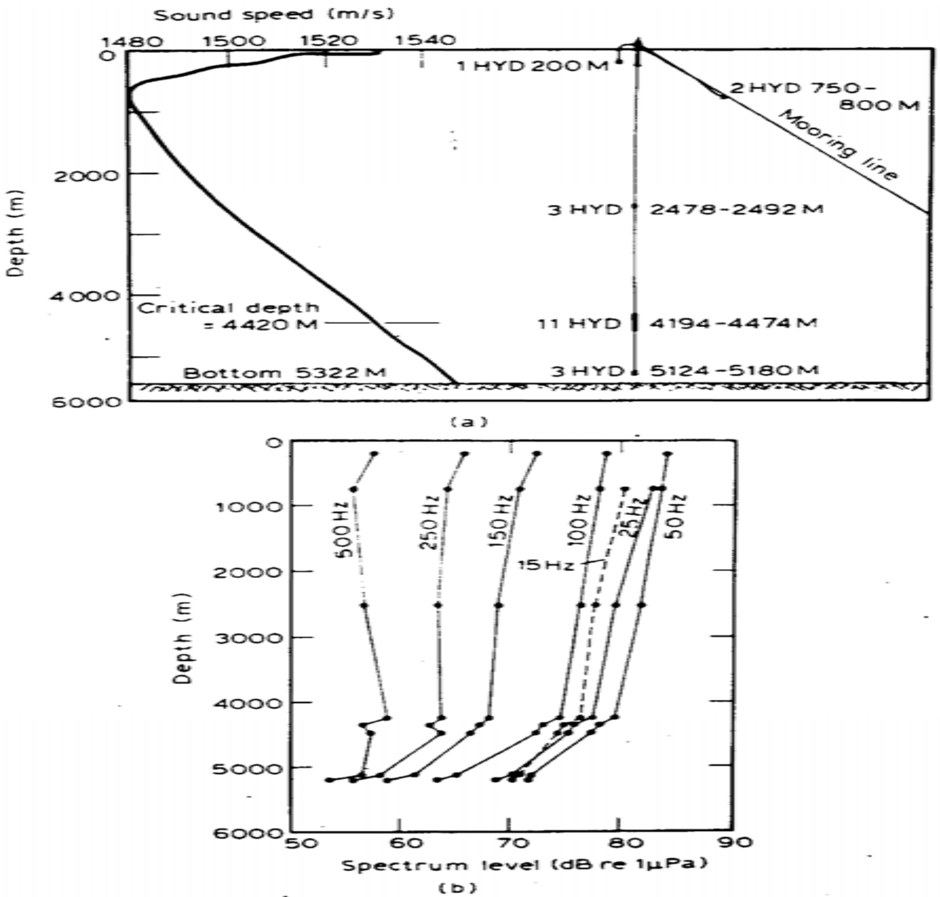


Illustration of the conversion of coastal shipping noise, represented by high-angle rays, to noise in the deep sound channel, represented by horizontal rays (adapted from Wagstaff, 1981).



Bathymetric and sound speed structure in the North Pacific Ocean. The noise from distributed shipping sources at high latitudes can enter the sound channel and propagate with little attenuation to lower latitudes. Relationships between the sound speed structure and the prevailing water masses are also illustrated (Kibblewhite *et al.*, 1976).

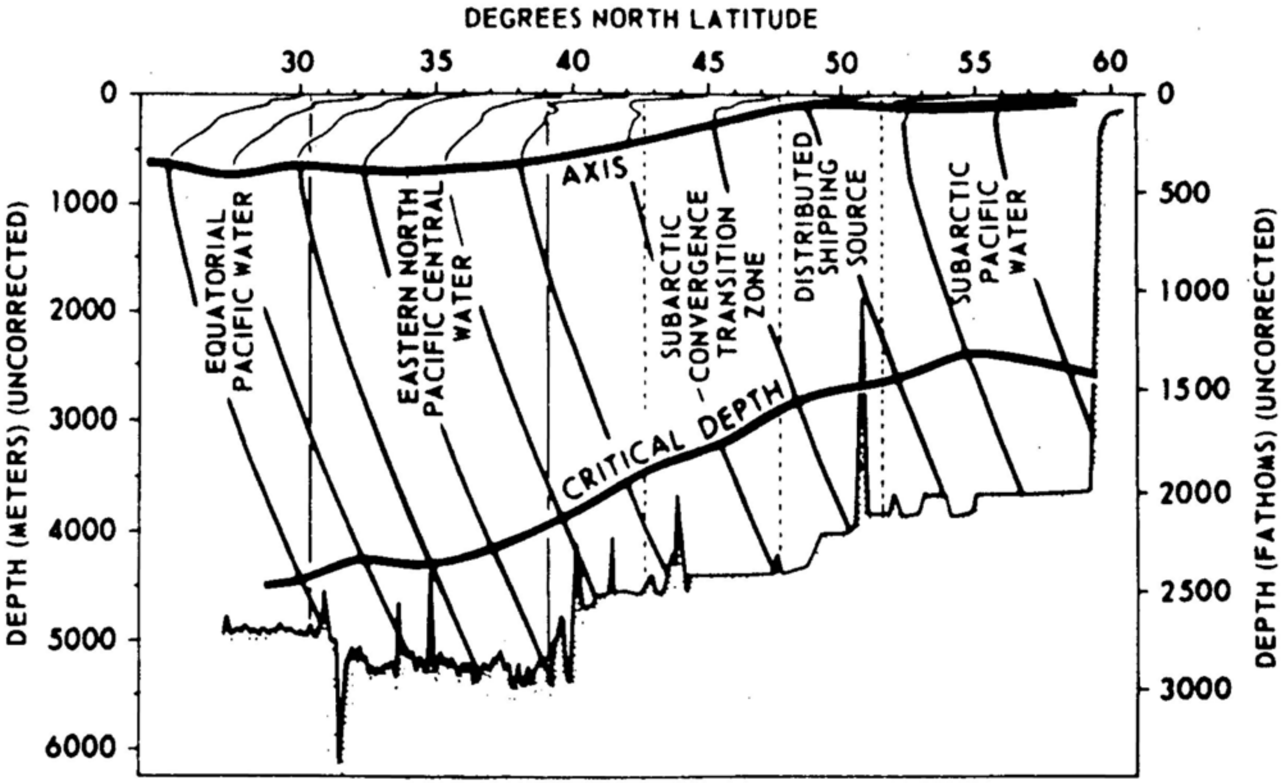
Ambient Noise Measurements



Northeast Pacific Ocean ambient noise measurements (Morris, 1978): (a) sound speed profile and hydrophone depths; (b) measured noise profiles.

Paul C. Etter, Underwater Acoustic Modeling, E & F SPON, New York, NY, Second Edition, 1996

Bathymetric and Sound Speed Structure (North Pacific)



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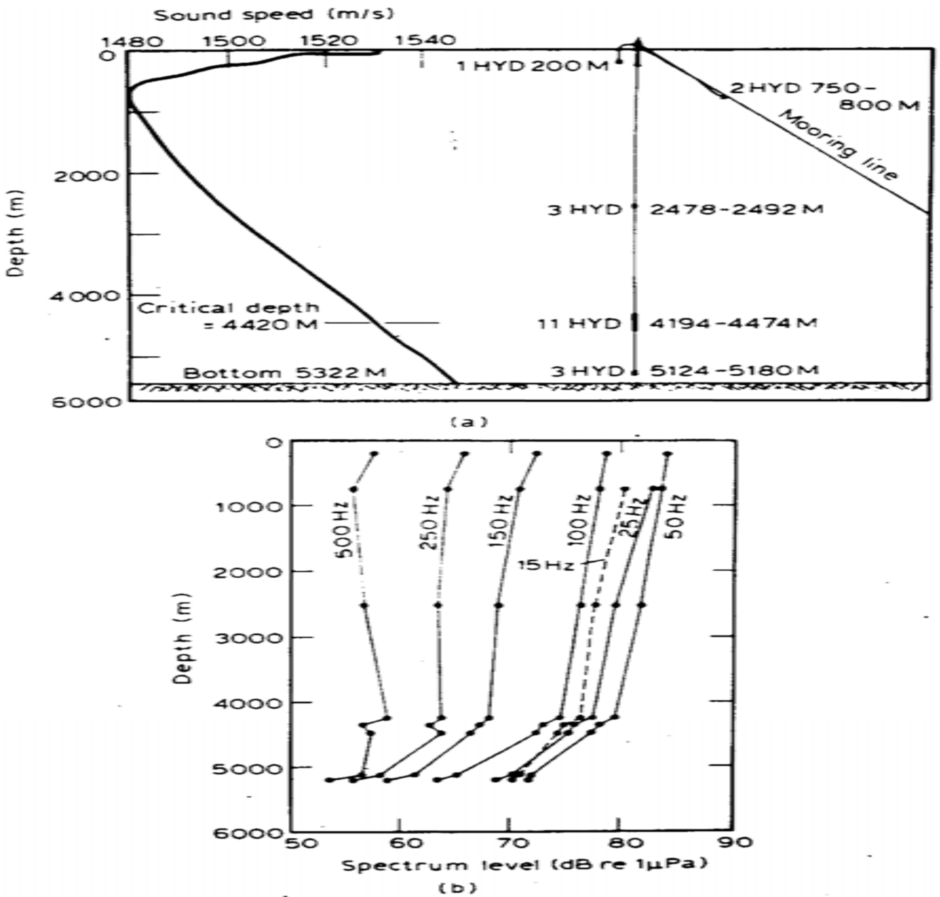

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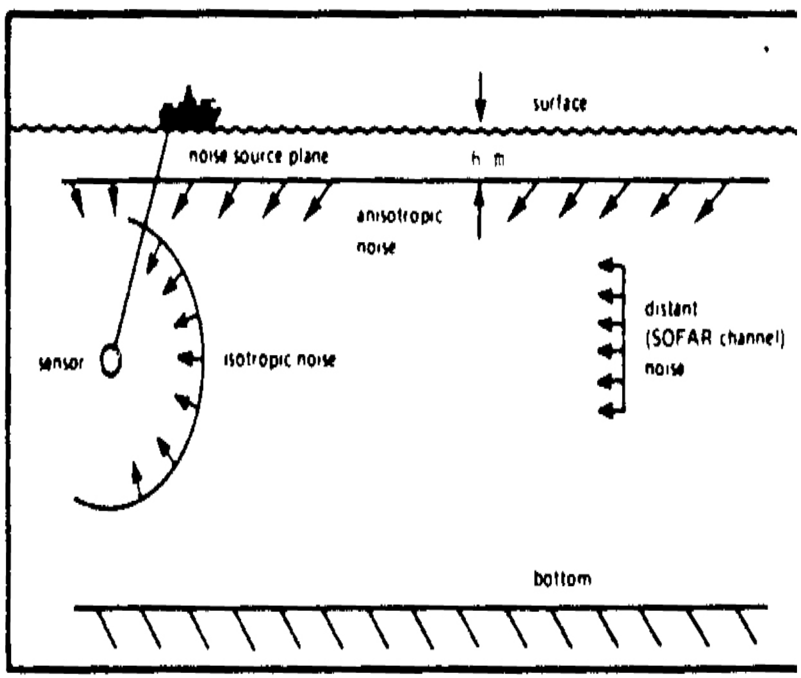
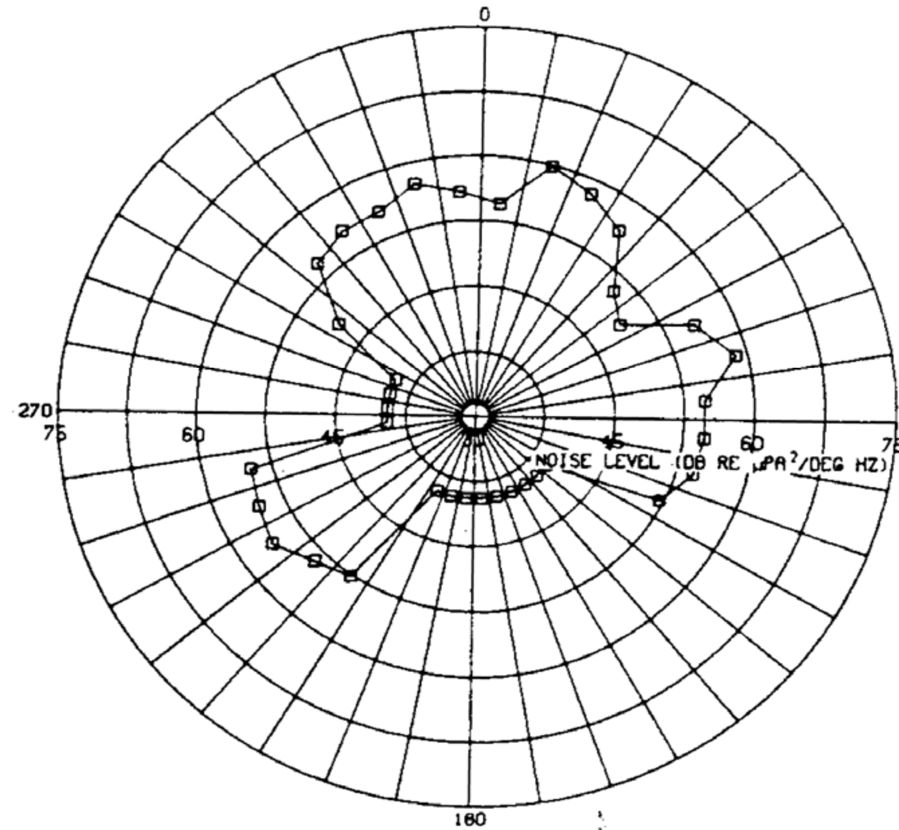
Directionality of Ambient Noise

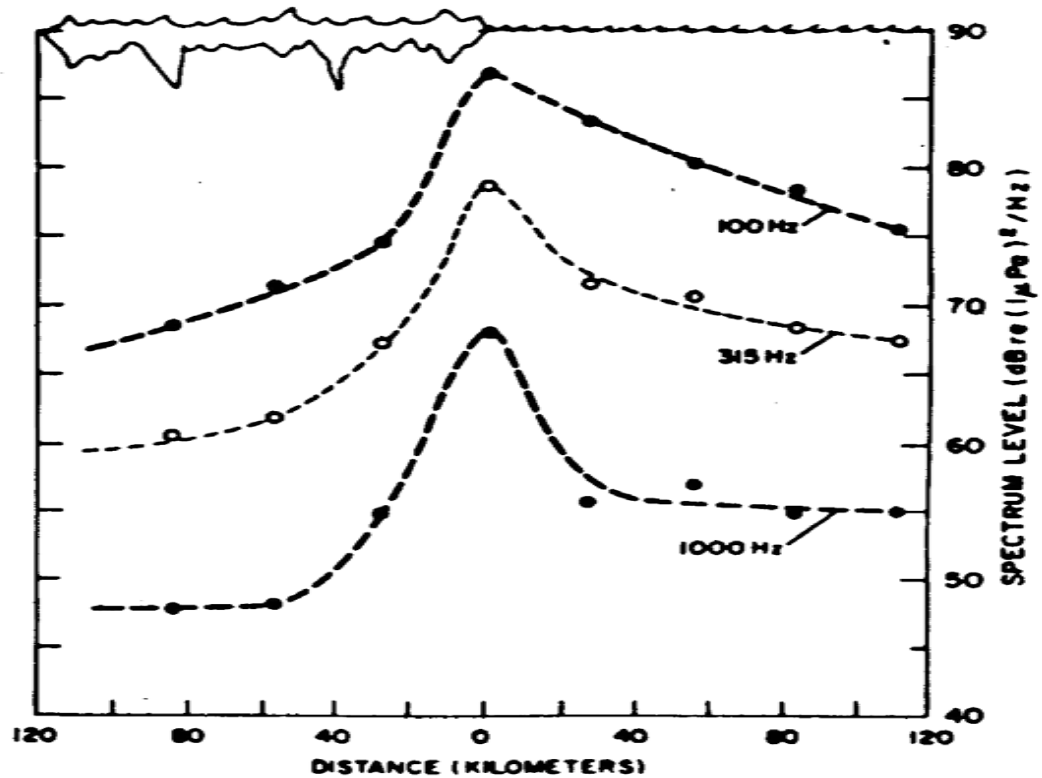


The noise field used in the RANDI model (Wagstaff, 1973).



Per-degree horizontal directionality of ambient noise in 10° sectors generated by the RANDI noise model for a frequency of 100 Hz at a depth of 91 m in the North Pacific Ocean (Wagstaff, 1973).

Ambient Noise Near Compact Ice Edge



Variation of median ambient noise sound pressure spectrum levels with distance from a compact ice edge for frequencies of 100, 315 and 1000 Hz in sea state 2 (Diachok, 1980).

Radar Equation Definitions

SNR	=	Signal-to-noise ratio
P_T	=	Transmitter power
G_T	=	Transmitter directional gain
r	=	Range to target
α	=	Attenuation coefficient
σ	=	Effective target scattering cross-section
A_R	=	Effective area of receiver
KBT	=	Thermal noise power
P_{amb}	=	Ambient noise power
P_{rev}	=	Reverberation backscattered power

Convert equation to dB by taking 10 log of both sides of the equation.

The Radar/Sonar Equation

SNR = $\frac{\text{Signal Power}}{\text{Noise Power}}$

$$\begin{array}{ccccc} \text{Source} & & \text{Target} & \text{Transmission} & \\ \text{Level} & & \text{Strength} & \text{Loss} & \end{array}$$

$$SNR = \frac{\left[\frac{P_T G_T}{4\pi}\right] \frac{1}{r^2} \cdot \boxed{\frac{\sigma}{4\pi}} \boxed{\frac{1}{r^2}} A_R}{kTB + P_{amb} + P_{rev}}$$

$$\text{where} \quad A_R = \frac{\lambda^2}{4\pi} G_{Receiver}$$

= Gain Against Noise or reverberation

Sonar Equations (dB Values)

Active Sonar – Ambient Noise Background

$$[SL - 2TL - + TS] = [NL – DT] + DT + SE$$

Active Sonar – Reverberation Background

$$[SL - 2TL - + TS] = (RL) + DT + SE$$

Passive Sonar

$$[SL - TL] = (NL – DI) + DT + SE$$

Signal Level =	**Effective Masking Noise**	**+**	**Required Signal to Noise Ratio**	**+**	**Signal Excess Relative to DT**

Solving the Sonar Equation

The sonar equation expresses the signal-to-noise ration as function of the sonar and environmental parameters. At the maximum detection range, the received signal power divided by the noise power is equal to the detection threshold. (Signal excess is zero.) Uncertainties in some of the parameters, especially TL and NL, limit the accuracy of prediction to a few dB.

Figure of Merit

FOM equals the maximum allowable one-way transmission loss in passive sonar or the maximum allowable two-way transmission in active sonar.

Passive FOM = TL = SL - [NL – DI + DT]

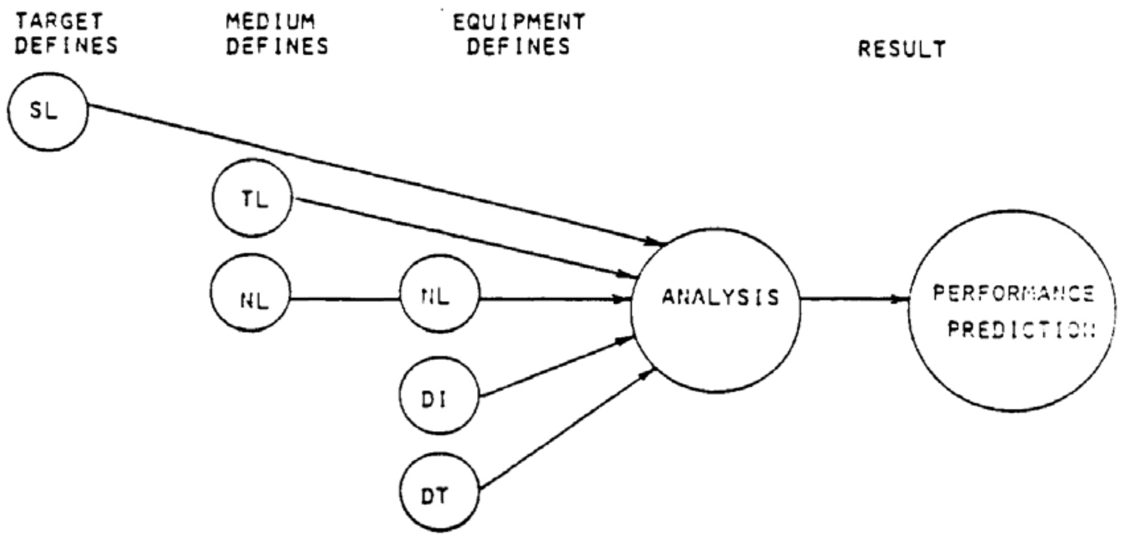
Active FOM = 2TL = SL + TS – [NL – DI + DT]

<u>Note:</u>

Including TS in active FOM disagrees with Urick, but is a common practice.

FOM is not useful for reverberation limited ranges.

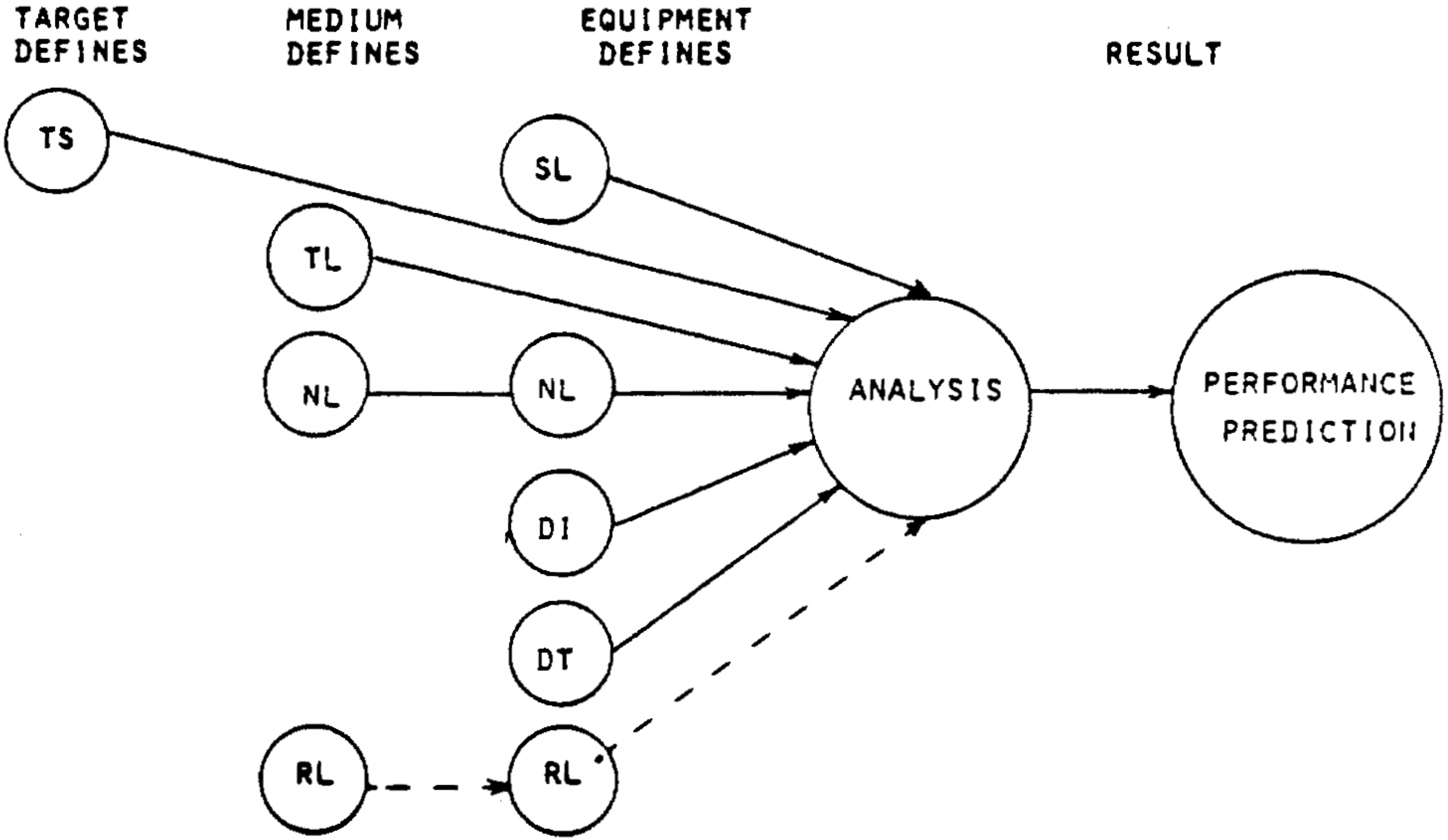
Passive Sonar Prediction



[SL – TL]	-	[NL – DI]	=	DT
Received Signal Level	-	Background Masking Noise	=	Detection Threshold

Urick [75]

Active Sonar Prediction



Urick [75]

Active Sonar Parameters

Parameter	Symbol	Important Factors
Source Level	SL	Input Power Conversion Efficiency Directivity Index Cavitation Limitations
Target Strength	TS	Target Area Target Aspect Pulse Duration Frequency
Reverberation Level	RL	Source Level Beam Width Pulse Length Boundaries Scattering Layers Frequency

Detection Threshold

$$DT = 10 \log \frac{\text{Req Signal Power for a Spec. Performance}}{\text{Noise Power per Hz at the Receiver Input}}$$

Detection Threshold (DT) is defined as the ratio (in decibels) of the signal power in the receiver bandwidth to the noise ower spectrum level (in a 1 Hz band) measured at the receiver terminals, required for detection at some specified level of correctness of the detection decision [i.e., p(D) and p(FA)].

Typical requirements per decision (or per look):

p(D) = 0.5 or 0.9

p(FA) = 10-6

Urick [75]

Detection Decision

Decide "Signal Present" when the input exceeds the expected value of noise by more than a bias level; otherwise, decide "Signal Absent."

Decision / True Input	Signal Present	Signal Absent
Signal Present	Correct DETECTION p(D)	False Dismissal 1-p(D)
Signal Absent	FALSE Alarm p(FA)	Correct Dismissal 1-p(FA)

Urick [75]

Signal Detection Experiment

GAUSSIAM NOISE GENERATOR

Average noise level N

15-second sections

STEADY SINUSOIDAL SIGNALS

Duration T

Average Power S

THE EXPERIMENT

Signals embedded in half the 15-second noise sections

(or echo cycles)

Listeners push button when “signal present”

LISTENER TRAINING

Easily detected signals

Signal levels gradually reduced

Data Collection

- "HIT" SCORED WHEN LISTENER RESPONDED WITHIN 1 SEC. AFTER SIGNAL OCCURRENCE

- "FALSE ALARM" SCORED WHEN LISTENER RESPONDED TO NOISE

- LISTENERS WERE NOT RESTRICTED TO ONE RESPONSE PER NOISE SECTION

- SCORES WERE TABULATED USING LAST RESPONSE IN EACH SECTION

Estimates of P_D and P_{FA}

$$P_D = \frac{\text{NUMBER OF “HITS”}}{\text{NUMBER OF NOISE SECTIONS WITH SIGNALS}}$$

$$P_{FA} = \frac{\text{NUMBER OF “EMPTY SECTIONS” IN WHICH RESPONSES WERE MADE}}{\text{NUMBER OF “EMPTY” NOISE SECTIONS}}$$

“EMPTY” NOISE SECTION IS NOISE SECTION WITH NO EMBEDDED SIGNAL.

Experiment Results

♦ LISTENED TO SEVERAL HUNDRED NOISE SECTIONS

- Different days
- Different instructions on strictness about responding

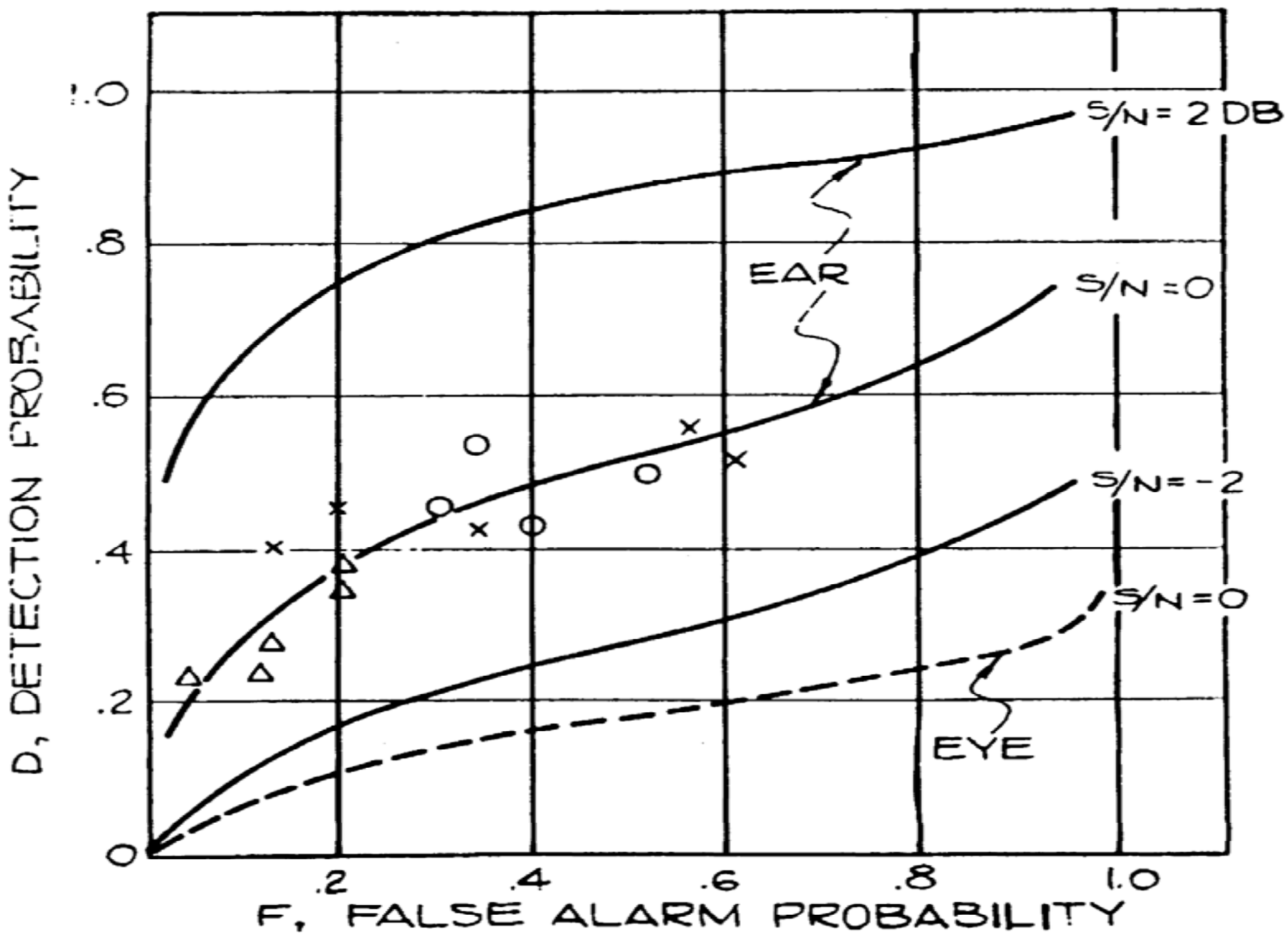
♦ P_D PLOT AGAINST P_{FA} GROUPED AROUND THEORETICALLY COMPUTED $\frac{S}{N}$ = 0 dB Curve

$$\frac{S}{N}(\text{dB}) = 10 \log_{10}\left[\frac{\text{Average Signal Power (watts)}}{\text{Average Noise Power (watts)}}\right]$$

♦ WHEN AVERAGE S/N CHANGED TO ±2 dB, MEASUREMENTS CLUSTERED AROUND S/N = ±2dB CURVES, ETC.

NOTE: $\log_{10}(1) = 0$ means $\langle S \rangle = \langle N \rangle$

Receiving Operating Characteristics (ROC) Operating Curves

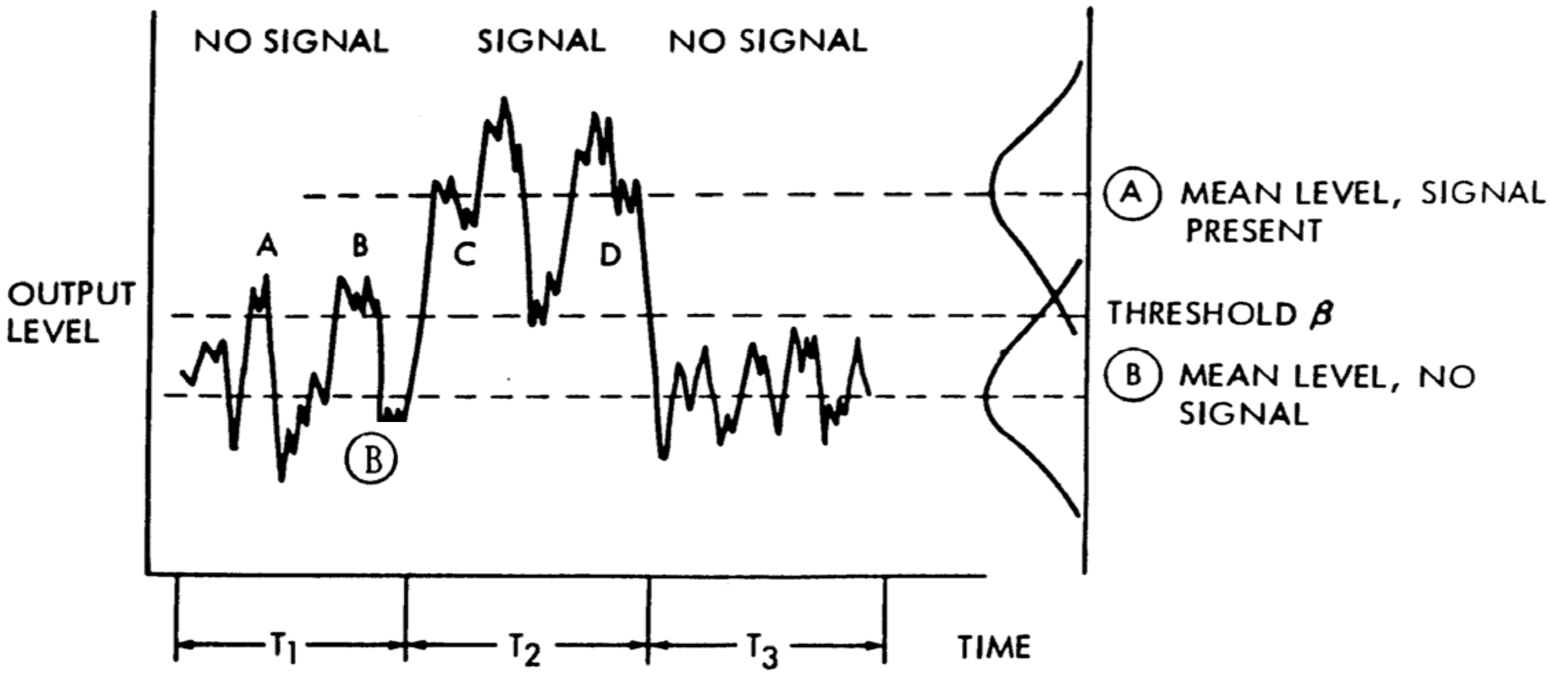


o Observer 1
x Observer 2
Δ Observer 3

Interpretation of Results

- P_D increases with S/N
- P_{FA} decreases as S/N increases
- Visual detection
  - Noise sections written out as pen recordings (such as time bearing plots)
  - Observers detect signal by viewing recorder plots
  - Different set of P_D vs. P_{FA} curves
  - Eye not as good (for detection) as ead

Concept Leading to Construction of ROC Curves



Output Levels:

•**Energy detector – Mean square amplitude of power**

•**Voltmeter**

- •**Oscillate about (B) when no signal present**
- •**Oscillate about (A) when signal present**

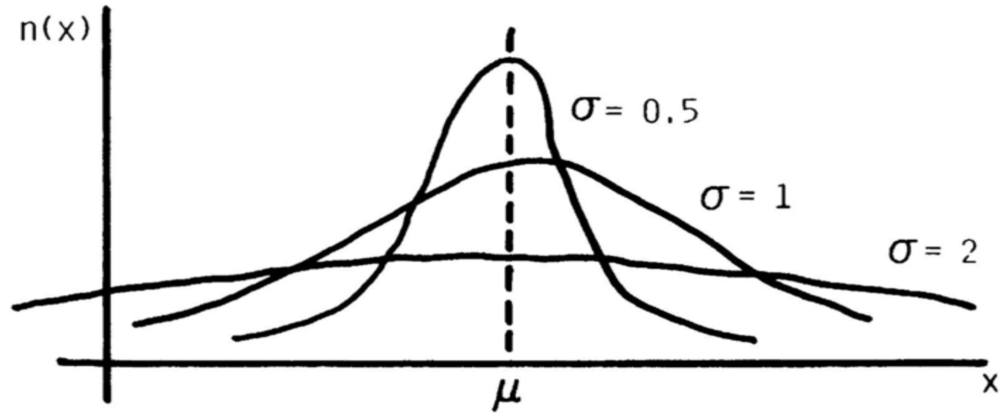
Gaussian Distribution (Model of Random Noise)

GAUSSIAN PROBABILITY DENSITY FUNCTION:

$$n(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

$$y = \frac{x-\mu}{\sigma}$$

$$n(y) = \frac{1}{\sqrt{2\pi}} e^{\frac{-y^2}{2}}$$



μ = mean

σ = standard deviation

$\mu \pm \sigma$: 68%

$\mu \pm 2\sigma$: 95%

$\mu \pm 3\sigma$: 99.7%

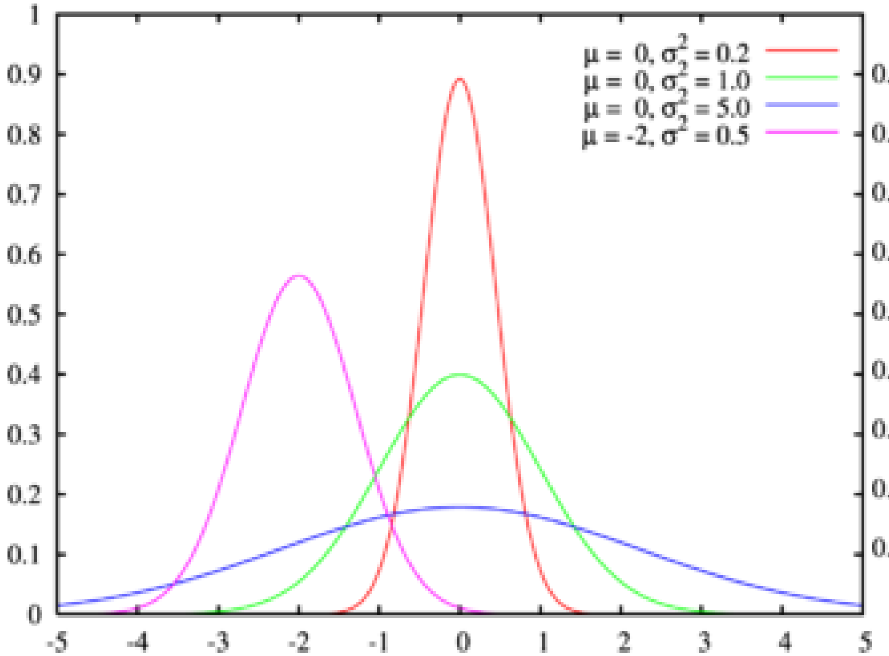
$\mu \pm .5\sigma$: 38%

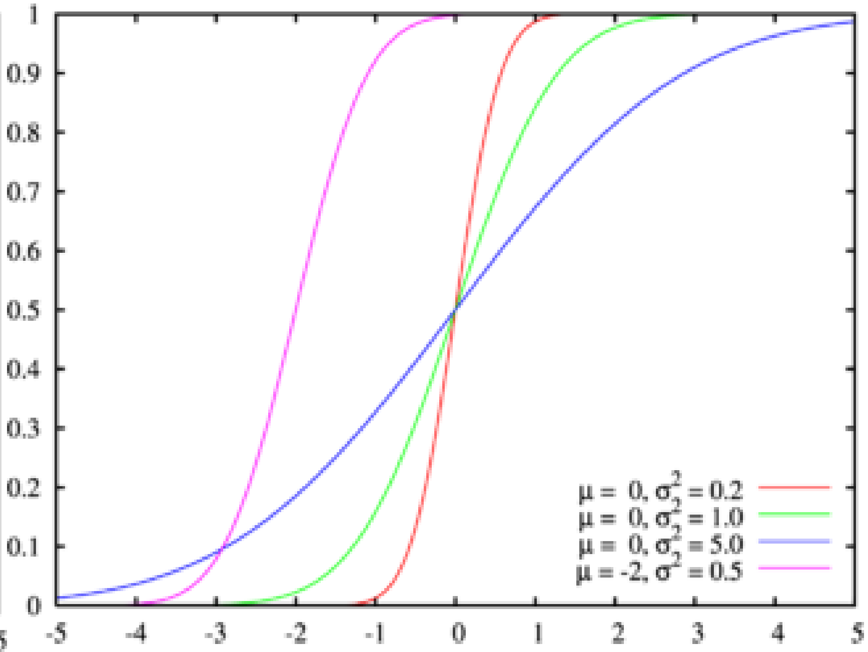
$\mu \pm 1.5\sigma$: 87%

$$F(y) = \int_{-\infty}^{y} \frac{e^{-\frac{t^2}{2}}}{\sqrt{2\pi}}\,dt$$

Central Limit Theorem: The random selection of values from any distribution (Gaussian or not) tends to a Gaussian distribution with the same mean and standard deviation as was in the original distribution.

Normal Distribution Probability Density and Cumulative Distribution Functions





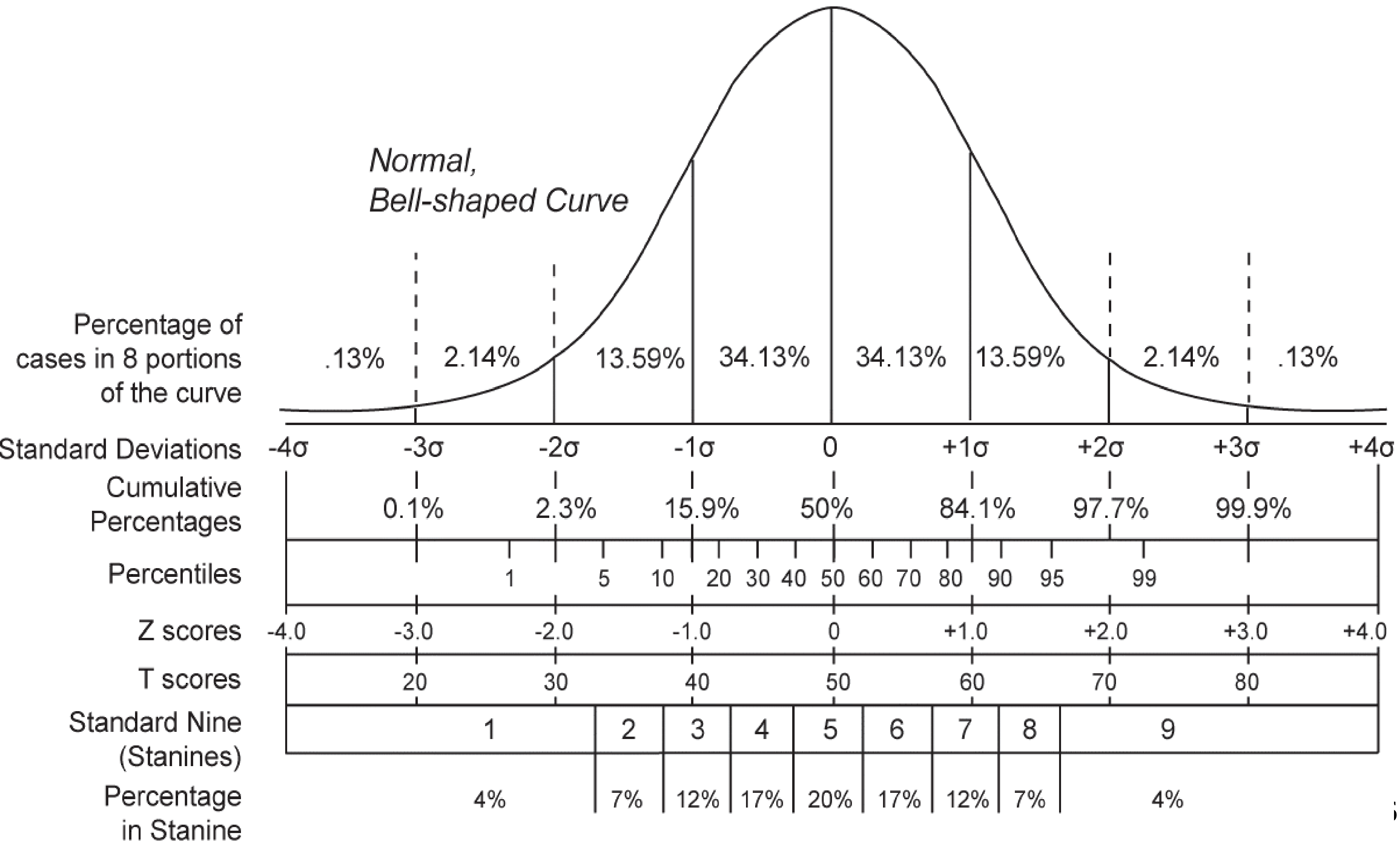
Normal Distribution Density

$$\frac{1}{\sigma\sqrt{2\pi}}\ \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

Cumulative Distribution Function

$$\frac{1}{2}\left(1+\operatorname{erf}\frac{x-\mu}{\sigma\sqrt{2}}\right)$$

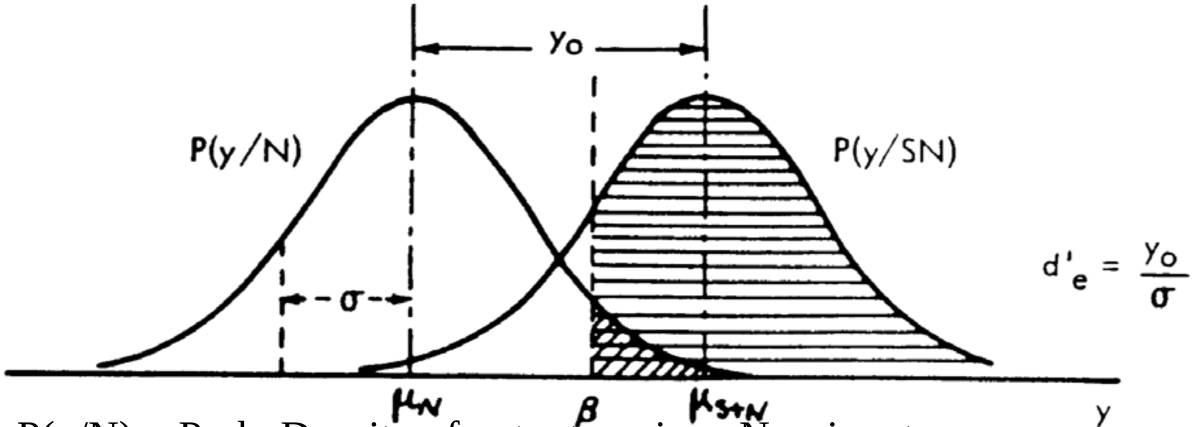
Normal Bell-Shaped Curve



Computing ROC Curves

- **DETECTABILITY INDEX:**

$$d'_e = \frac{\mu_{S+N} - \mu_N}{\sigma} = \frac{y_0}{\sigma}$$



P(y/N) = Prob. Density of output y, given N as input
P(y/SN) = Prob. Density of output y, given S+N as input

- IDEAL ENERGY DETECTOR

$$d'_e = \frac{E}{N_0} \cdot \frac{1}{(WT)^{1/2}} = \frac{S}{N} \cdot (WT)^{1/2}$$

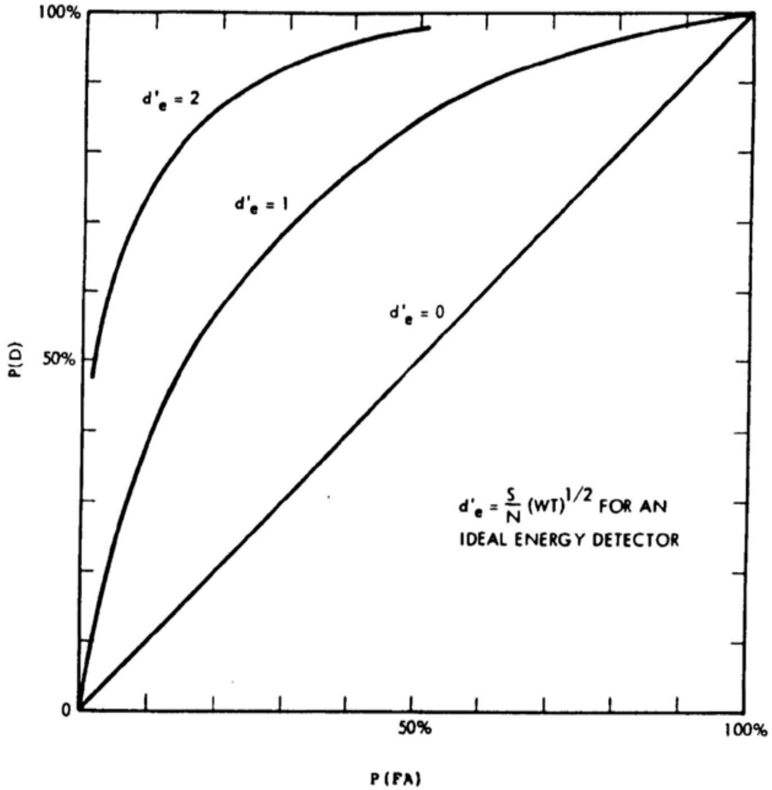
E = Total signal energy received during time T in band W
S = Signal power in band W (note: E = ST)
T = Sample time of detector
W = Input bandwidth of the detector system

Evaluation of P(D) and P(F)

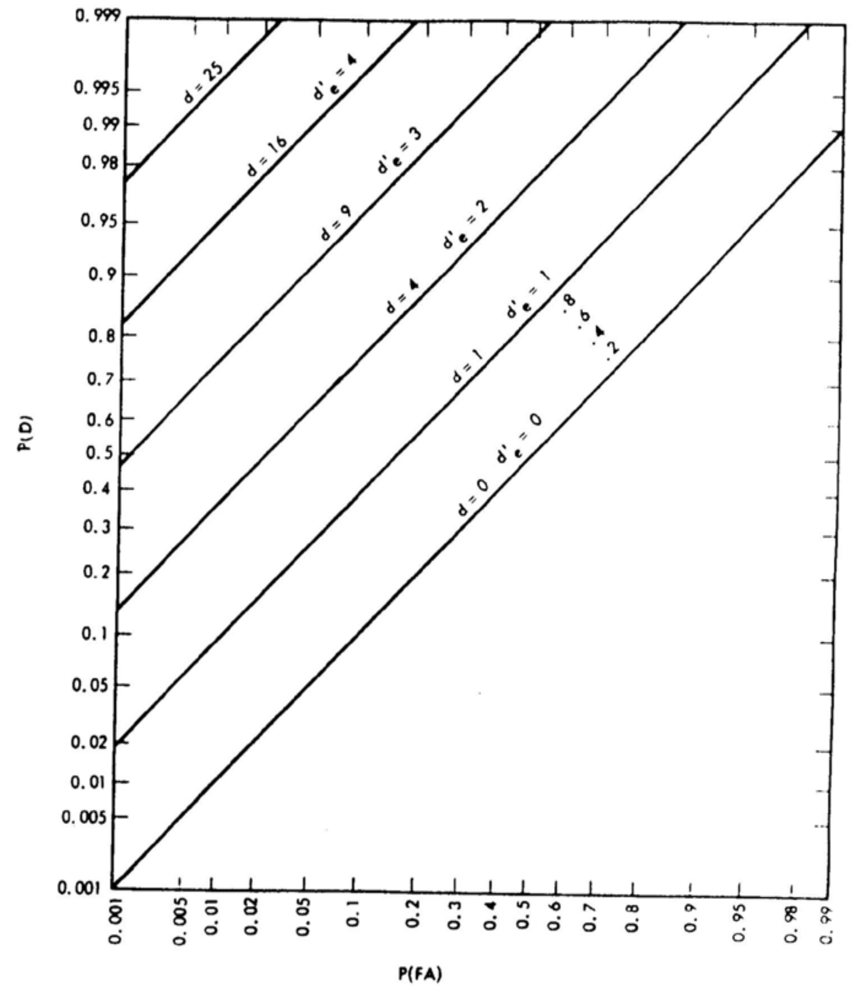
$$P(FA) = \int_{\beta}^{\infty} P_N(Y)\, dY = \int_{\beta}^{\infty} \text{(Prob. density of noise)}$$

$$P(D) = \int_{\beta}^{\infty} P_{SN}(Y)\, dY = \int_{\beta}^{\infty} \text{(Prob. density of signal and noise)}$$

ROC Curves for an Energy Detector



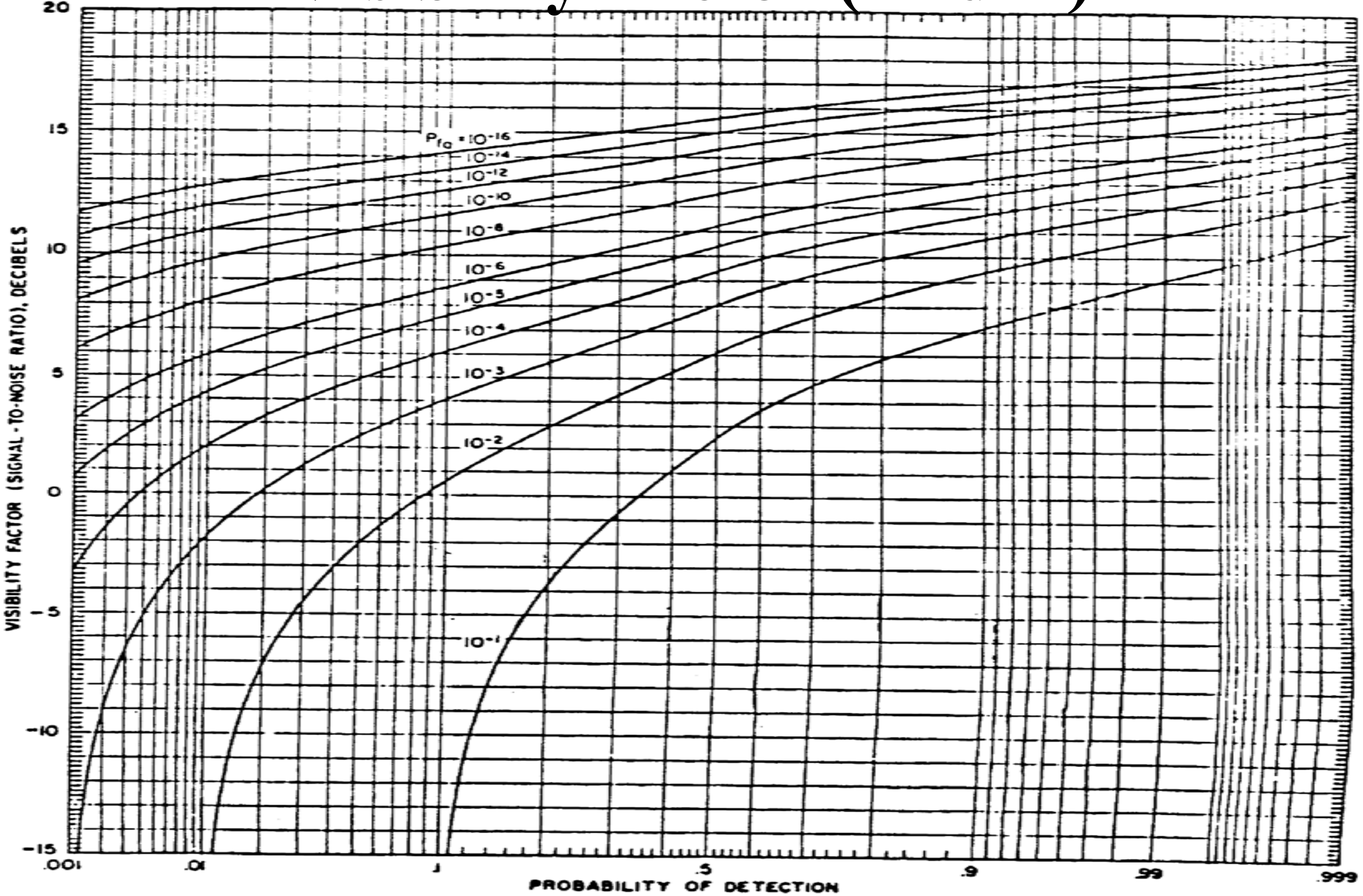
ROC CURVES FOR AN ENERGY DETECTOR. P(D) IS DETECTION PROBABILITY. P(FA) IS FALSE ALARM PROBABILITY. d'_e IS THE DETECTABILITY INDEX. S AND N ARE INPUT SIGNAL AND NOISE POWER IN THE DETECTOR BAND-WIDTH W. T IS THE SAMPLE TIME OF THE DETECTOR.



NORMAL ROC CURVES.

$d = (d'_e)^2 \simeq$ Output S/N

Visibility Factor (Radar)



Required signal-to-noise (visibility factor) at the input terminals of a linear-rectifier detector as a function of probability of detection for a single pulse, with the false-alarm probability (P_{FA}) as a parameter, calculated for a non-fluctuating signal
Blake [69]