Target Strength

When an active sonar pulse is transmitted into the water, some of the sound reflects off of the target. The ratio of the intensity of the reflected wave at a distance of 1 yard to the incident sound wave (in decibels) is the target strength, TS.

$$TS = 10\log\left(\frac{I_r}{I_i}\right) = 10\log\left[\frac{\sigma}{4\pi}\right]$$

$I_r \equiv$ Intensity reflected from target

$I_i \equiv$ Intensity incident on target

$\sigma \equiv$ Backscattering cross-section

I_r depends on the physical characteristics of the target and characteristics of the signal (angle and frequency). The result in the square brackets comes from the fact that if all the energy reflects from the target, the Power striking the target and the power leaving the target must be equal.

$$I_i\sigma = 4\pi r^2 I_r$$

The ratio of reflected to incident intensity is simply

$$\frac{I_r}{I_i} = \frac{\sigma}{4\pi r^2}$$

where r is 1 yard. The backscatter cross section is a number that represents the degree to which sound is scattered off a target. It is related to the size, shape and reflectivity of a target.

Can the quantity, target strength be solved for analytically? Yes, but only for simple geometric shaped objects. We will present how this can be done for a convex object and a simple sphere. For more complicated geometric objects, I have included a table from Urick, **Principles of Underwater Sound**, which gives the formula to calculate the target strength for many other shaped objects. For any irregularly shaped object, we may be able to model them as a simple geometric object but for a precise value, we would have to use empirical data.

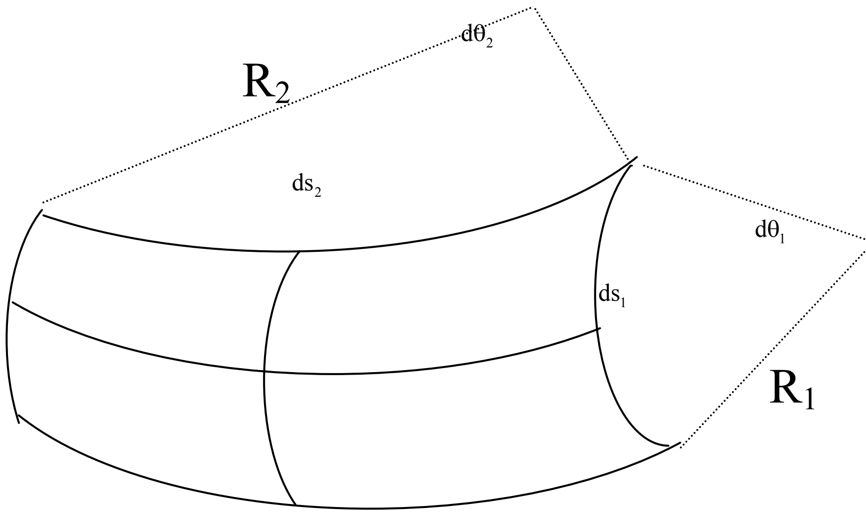
For analysis, assume that the incident wave is a plane wave (valid if source far from target) and that the scattered wave is spherical originating from the target. I_r is measured 1 yd (or 1 m) from the target.

Target Strength of an arbitrary convex object

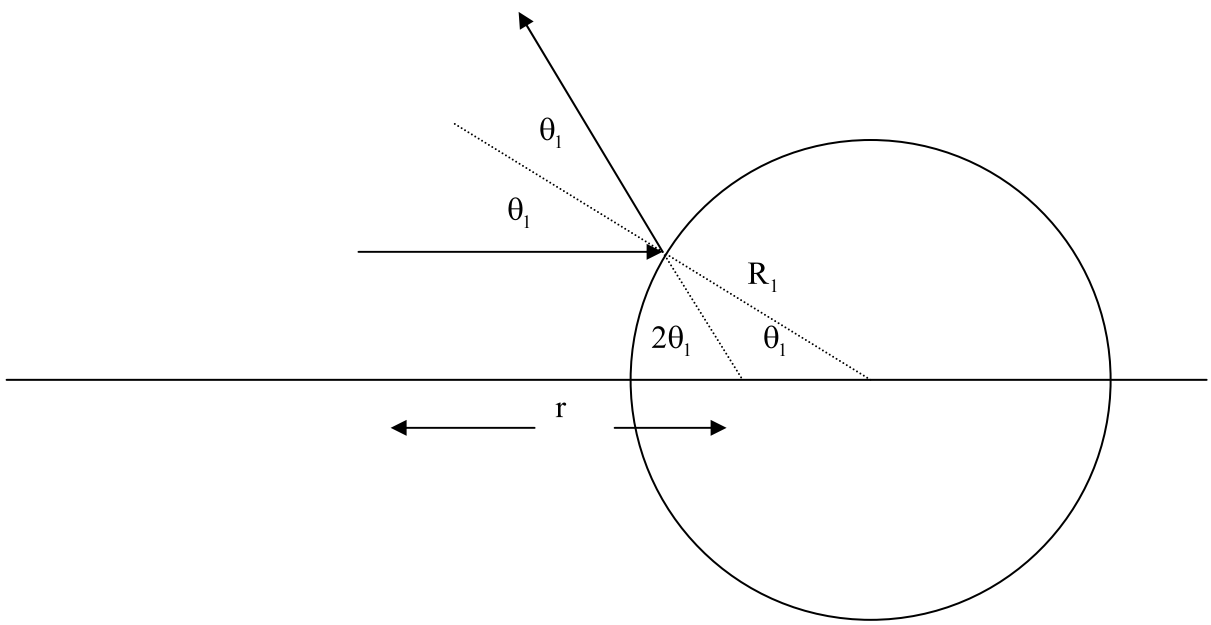
In the diagram below, let the surface area of the arbitrary convex surface be $dA=ds_1ds_2$. If the sound incident on the surface has an intensity, I_i, then the power striking the surface is

$$dP = I_i ds_1 ds_2 = I_i R_1 d\theta_1 R_2 d\theta_2$$

since $ds=Rd\theta$. The centers of curvature for the two sides of the surface are not in general the same point.



The pivotal question when examining the reflected intensity is what angles $d\theta_1$' and $d\theta_2$' does the sound energy bounce of the surface into. Examination of the ray diagram below shows that sound hitting the surface within an angle, θ_1, of the equator, bounce of the surface following the law of reflection. As such the ray departs the surface with an angle, $2\theta_1$, twice the incident angle. We notice that the exiting rays appear to emanate from a point half way between the center of curvature and the surface. In General Physics we called this a "focal point" and for a spherical mirror we recall that it was located at one half the radius of curvature.



With this in mind, we identify the surface the energy leaving the surface must pass through is

$$dA = ds_1' ds_2' = r2d\theta_1 r2d\theta_2 = 4r^2 d\theta_1 d\theta_2$$

The reflected intensity is then:

$$I_r = \frac{dP}{dA} = \frac{I_i R_1 d\theta_1 R_2 d\theta_2}{4r^2 d\theta_1 d\theta_2} = \frac{I_i R_1 R_2}{4r^2}$$

The resulting Target Strength follows from the definition:

$$TS = 10\log\left(\frac{I_r}{I_i}\right) = 10\log\left(\frac{\frac{I_i R_1 R_2}{4r^2}}{I_i}\right) = 10\log\left(\frac{R_1 R_2}{4r^2}\right)_{r=1yd} = 10\log\left(\frac{R_1 R_2}{4}\right)$$

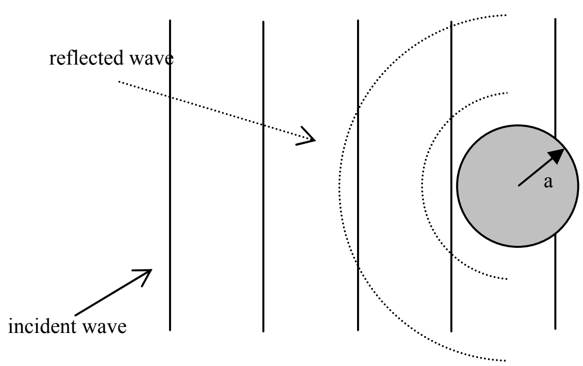
As a special case, let us look at a simple rigid sphere. In this case, $R_1=R_2=a$, the radius of the sphere. The Target Strength then becomes

$$TS = 10\log\left(\frac{a^2}{4}\right)$$

Let's see if this result makes physical sense.

Target Strength of Simple Rigid Sphere

Case I: (ka>>1 {or ka > 10} or in other words, when the radius of the sphere is much larger than the wavelength of the incident wave.)



If the rigid sphere is large compared with the wavelength of the incident sound wave and the sphere is an isotropic reflector (reflects sound equally in all directions), we can use the diagram at right:

The power of the incident wave that will be reflected is that power of the wave incident on a cross-section of the sphere where:

$$P_i = I_i \pi a^2$$
$$\text{where } \sigma = \pi a^2$$

Since the power of the incident wave is all reflected back, we find that:

$$\text{power reflected} = \text{power incident}$$

$$P_r = P_i$$

$$I_T 4\pi r^2 = I_i \pi a^2$$

$$\frac{I_T}{I_i} = \frac{a^2}{4r^2}$$

Then using the definition of target strength, we find:

$$TS = 10\log\frac{I_T}{I_i} = 10\log\frac{a^2}{4r^2}\bigg|_{r=1\,yd}$$

$$TS = 10\log\frac{a^2}{4}$$

This is exactly the same result we obtained above as a special case of an arbitrary convex surface. Note that the above target strength result is independent of frequency (as long as ka>10). Target strength just depends on the radius, a. For a 1 cm radius rigid sphere, $\sigma_{bs} = 2.5\times10^{-5}\ m^2$ and TS=-46 dB. A 2 m radius sphere however would have a TS= 0 dB. This simple approximation is only meaningful for high frequencies where the wave effects can be averaged. For lower frequencies (longer wavelengths), the wave effects must be taken into account.

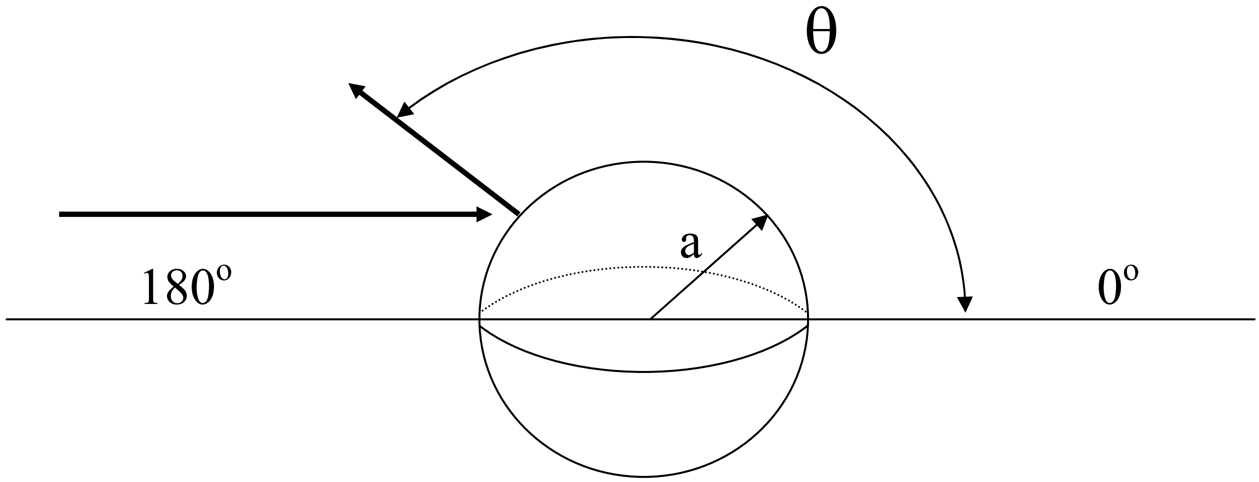
Case II: (ka<1)

When the wavelength of the incident wave is large compared to the size of the sphere, some of the wave will appear to continue past the ball as if it did not exist. There will actually be very little backscattering. This case, Lord Rayleigh showed that:

$$\frac{I_T}{I_i} = \frac{\pi^2 V^2}{\lambda^4 r^2}\left[\frac{3}{2}\cos\theta - 1\right]^2$$

where:

$V \equiv$ volume of the sphere



For the target strength, μ=-1 (cos 180° = -1, straight backscatter) and r = 1 yd. The above then becomes:

$$TS = 10\log\left\{(ka)^4\left(\frac{25}{36}\right)[a]^2\right\}$$

$$\Rightarrow \sigma_{bs} = 4\pi\frac{25}{36}k^4a^6$$

For Case I, one of the major assumptions was that the entire cross-sectional area (σ) contributed to the backscattering of the incident sound energy. For this case, the ratio of the effective backscattering cross-section to the geometric cross-section would be:

$$\frac{\sigma_{bs}}{\pi a^2} = 2.8(ka)^4$$

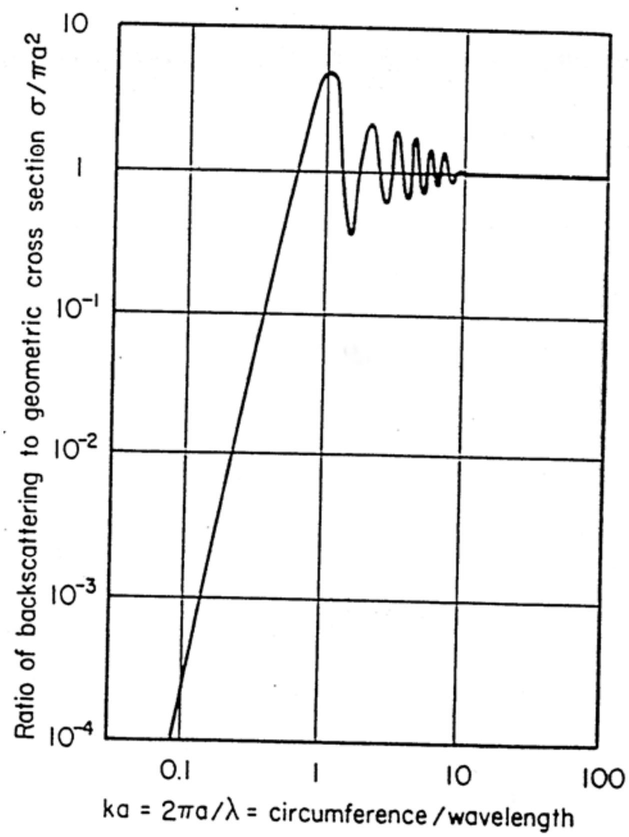
Notice that $\sigma/\pi a^2$ increases very rapidly with frequency $\left(\propto f^4\right)$, therefore target is barely detectable when size is much smaller than the wavelength. As frequency increases there is a limit to Rayleigh scattering:

$$ka = \frac{2\pi a}{\lambda} = 1$$

Occurs when $\lambda = 2\pi a$

Case III: If 1<ka<10

For this exceptional case, we can use the plot given below which was taken from Urick, **Principles of Underwater Sound**, p. 299. This plot shows the ratio of the backscattering cross-section to the geometric cross-section as a function of ka, which can be used to calculate a value for the target strength. Target response in this range is dominated by interference between reflected wave and "creeping waves" refracted around the surface of the sphere.



Fluid Sphere

Spherical target is no longer ideally rigid, therefore in the Rayleigh regime:

$$\sigma_{bs} = k^4 a^6 \left[\frac{1}{3} - \frac{\rho_1 c_1^2}{3\rho_2 c_2^2} + \frac{\rho_2 - \rho_1}{2\rho_2 + \rho_1} \right]^2$$

$\rho_1, c_1 \equiv$ density and sound speed in water

$\rho_2, c_2 \equiv$ density and sound speed in target

When $\rho_2 > \rho_1$ and $c_2 > c_1$, therefore σ_{bs} approaches that of ideal rigid sphere. When $\rho_2 < \rho_1$ and $c_2 < c_1$, σ_{bs} is dominated by the compressibility of the sphere:

$$\sigma_{bs} = k^4 a^6 \left[\frac{\rho_1 c_1^2}{3\rho_2 c_2^2} \right]^2$$

σ_{bs} is much higher than for a rigid sphere of the identical radius. For example, the target strength of an air bubble is 75 dB higher than the target strength of rigid sphere with same radius.

Scattered Gas Bubbles

Backscatter of gas bubbles in sea water is widely studied because of the important acoustic implications. Air bubble clouds can create undesirable reverberation from the sea surface. Gas bubbles are also present in sediment and are an essential component of seafloor backscattering. Effects of random populations on the acoustic propagation and backscattering are difficult to predict accurately other than statistically. Gas bubble acoustic behavior is dominated by resonance. For frequencies near the resonance frequency (f_0 depends on bubble size), backscattering and absorption are enhanced;

$$\sigma_{bs} = \frac{a^3}{\left(\left(\frac{f_0}{f}\right)^2 - 1\right)^2 + \delta^2}$$

$$f_0 \equiv \text{ resonant frequency}$$

$$\delta \equiv \text{ damping term}$$

Resonant frequency can be approximated as:

$$f_0 = \frac{1}{2\pi a}\sqrt{\frac{3\gamma P_w}{\rho_w}} \approx \frac{3.25}{a}\sqrt{1+0.1z}$$

$$\rho_w = 1000 \text{ kg/m}^3$$

$$P_w \equiv \text{ hydrostatic pressure in Pa } \left(\approx 10^5\left(1+0.1z\right)\right)$$

$$z \equiv \text{ depth in meters}$$

$$\gamma \equiv \text{ adiabatic constant for air } (\approx 1.4)$$

Damping effect is due to the combined effects of radiation, shear viscosity and thermal conductivity. A good approximation is $\delta \approx 0.03\, f_k^{0.3}$ for $1 \text{ kHz} < f_k < 100 \text{ kHz}$, where f_k is the frequency in kHz.

Fish Target Strength

Main contribution for fish target strength comes from the swim bladder. This gas-filled bladder shows a very strong impedance contrast with the water and fish tissues. It behaves either as a resonator (frequencies of 500 Hz-2 kHz depending on fish size and depth) or as a geometric reflector (> 2 kHz). This swim bladder behaves very similar to gas bubbles. The difference in target strength between fish with and without swim bladder can be 10-15 dB.
A semi-empirical model most often used is:

$$TS_{fish} = 19.1 \log L + 0.9 \log f_k - 24.9$$

Love (1978)

This formula is valid for dorsal echoes at wavelengths smaller than fish length L.
A more detailed model is:

$$TS_{fish} = 20\log L - TS_{spec}$$

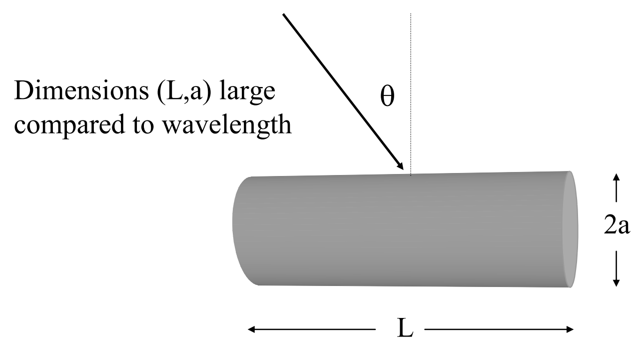
McLennan and Simmonds (1992)

TS_{spec} is given in Table 3.1 of Lurton, p.77. Note the lowest TS_{spec} is for mackerel which has no swim bladder. As frequencies approach the resonant frequency around 1 kHz, the target strength increases and can reach -25 to -20 dB.

For many other geometric shapes:

Use the tables given at the end of this lesson. Below are the equations and definition of terms for a cylinder.

Scattering from Cylinders



$$TS = 10\log\left[\left(\frac{aL^2}{2\lambda}\right)\left(\frac{\sin\alpha}{\alpha}\right)^2 \frac{\cos^2\theta}{(1\text{yd})^2}\right]$$

$$\alpha = \frac{2\pi L}{\lambda}\sin\theta$$

$\theta = 0^o$ ⇒

$$TS = 10\log\left[\left(\frac{aL^2}{2\lambda}\right)\frac{1}{(1\text{yd})^2}\right]$$

Conclusion

One of the main points of this section is that it is extremely difficult to get an accurate value for the target strength of a complex target but, if we can approximate the target as a simple geometric shape, we can calculate a value that sould be sufficient.

For the wavelengths that we typically use for active sonar systems though, a rough approximation that can often be used is that the target strength will be directly related to the cross-sectional area of the target.

Form	*t* *TS=10log(t)*	*Symbols*	*Direction of incidence*	*Conditions*
Any convex surface	$\frac{a_1 a_2}{4}$	a_1a_2 = principal radii of curvature r = range k = 2π/wavelength	Normal to surface	ka_1, ka_2 >>1 r>a
Large Sphere	$\frac{a^2}{4}$	a = radius of sphere	Any	ka>>1 r>a
Small Sphere	$61.7\frac{V^2}{\lambda^4}$	V = vol. of sphere λ = wavelength	Any	ka<<1 kr>>1
Infinitely long thick cylinder	$\frac{ar}{2}$	a = radius of cylinder	Normal to axis of cylinder	ka>>1 r > a
Infinitely long thin cylinder	$\frac{9\pi^4 a^4}{\lambda^2} r$	a = radius of cylinder	Normal to axis of cylinder	ka<<1
Finite cylinder	$\frac{aL^2}{2\lambda}$	L = length of cylinder a = radius of cylinder	Normal to axis of cylinder	ka>>1 r > L^2/λ
	$\frac{aL^2\left(\sin\beta/\beta\right)^2 \cos^2\theta}{2\lambda}$	a = radius of cylinder β = kLsinθ	At angle θ with normal	
Infinite Plane surface	$\frac{r^2}{4}$		Normal to plane	
Rectangular Plate	$\left(\frac{ab}{\lambda}\right)^2\left(\frac{\sin\beta}{\beta}\right)^2 \cos^2\theta$	a,b = sides of rectangle β = ka sinθ	At angle θ to normal in plane containing side a	r > a^2/λ kb >> 1 a > b
Ellipsoid	$\left(\frac{bc}{2a}\right)^2$	a, b, c = semimajor axis of ellipsoid	parallel to axis of a	ka, kb, kc >>1 r >> a, b, c
Circular Plate	$\left(\frac{\pi a^2}{\lambda}\right)\left(\frac{2J_1(\beta)}{\beta}\right)^2 \cos^2\theta$	a = radius of plate β = 2kasinθ	At angle θ to normal	r > a^2/λ ka>>1
Circular Plate	$\left(\frac{4}{3\pi}\right)^2 k^4 a^6$	a = radius k = 2π//λ	Perpendicular to plate	ka<<1

Problems

1. Johns Hopkins Applied Physics Lab is researching active, mine mapping sonar. The sonar they are using uses a frequency of 40 kHz. The mines they are trying to detect are spherical balls that are 1.4 m in diameter.

Which of the following is true:

a) The TS of the mines can be approximated using the large sphere formula ($TS = 10\log\frac{a^2}{4}$) since ka>>1.
b) the TS of the mines can be approximated using the small sphere formula ($\mathrm{TS} = 10\log\left(61.7\frac{\mathrm{V}^2}{\lambda^4}\right)$) since ka<<1.
c) The target strength of the mines does not depend on the frequency of the sonar system.
d) Lower frequency sonar should be used to get better spatial resolution of the mines.

2. If the target strength of the mines in problem 1 is found to be –9.1 dB, what would be the intensity of a return wave if the incident wave had an intensity of 21 W/m^2?

3. What would be the best approximation of the target strength of a submarine that is 300 meters long, and 30 meters in diameter? Assume the frequency of the active sonar is 40 kHz.

4. Given a sphere of radius 1.0 m in water (c = 1500 m/s) for what range of frequencies is the sphere considered to be
   a) A “large perfectly rigid” sphere (corresponding to specular or geometrical scattering).
   b) A “small fixed rigid” sphere (corresponding to Rayleigh scattering).

5. A modern torpedo is roughly 65 cm in diameter and 6 m long. An active sonar of frequency 20 kHz is used to measure the target strength when c = 1500 m/s. For each case take r = 1000 m.
   a) Why is range, r, given in this problem?
   b) If from the beam aspect, we consider the torpedo to be a cylinder, what target strength is expected.
   c) If from head-on we take the nose to be spherical, what target strength is to be expected?

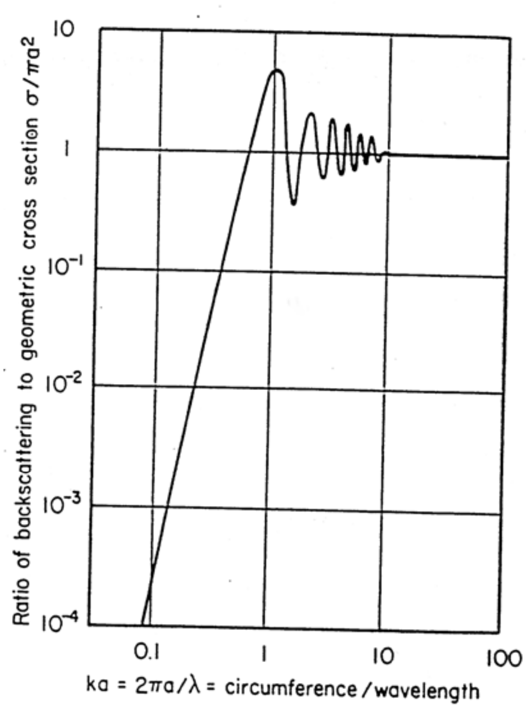
6. The first teardrop shaped submarine was USS Albacore, shown below at its museum site in Portsmouth NH.

Image courtesy of the Historic Naval Ships Association

Consider USS Albacore to be an ellipsoid of length 68 m and diameter 9.0 m at the midpoint. Calculate the target strength for active sonar at a beam aspect.

7. A sound beam of frequency 15 kHz is being used to search for a thick rectangular flat plate with dimensions 5.0 m x 3.0 m dropped from an oil rig at a depth of 100 m. Calculate the target strength of the plate:
   a) At normal incidence, and
   b) At an angle of 30^{o} from the normal in the plane of the longer axis of the plate.

8. Given a sphere of radius 0.20 m in seawater where c = 1500 m/s, use the below figure to determine:
   a) The ratio of backscattering to geometric cross section for 10 Hz, 100 Hz, 1000 Hz, 10 kHz.
   b) The target strength for frequencies of 10 Hz, 100 Hz, 1000 Hz, 10 kHz.



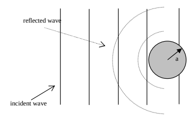
9. An acoustic pulse has an intensity of 10 W/m^2 incident 100 m from the center of an underwater target. The intensity of the 180^o reflected pulse has an average intensity of 3.16 $\mu W/m^2$ also measured 100 m from the target center. If spherical spreading is the only transmission loss, find the target strength of the object. Hint: EL = SL – 2 TS - +TS

Target Strength

$$TS = 10\log\left(\frac{I_r}{I_i}\right)$$

At r = 1 yd.

σ = scattering cross section



$$I_i\sigma = 4\pi r^2 I_r$$

$$TS = 10\log\left(\frac{\sigma}{4\pi r^2}\right) = 10\log\left(\frac{\sigma}{4\pi}\right)$$

Factors Determining Target Strength

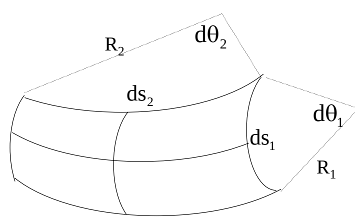
- the shape of the target
- the size of the target
- the construction of the walls of the target
- the wavelength of the incident sound
- the angle of incidence of the sound

Target Strength of a Convex Surface

Incident Power

$$dP = I_i ds_1 ds_2$$

$$dP = I_i R_1 d\theta_1 R_2 d\theta_2$$



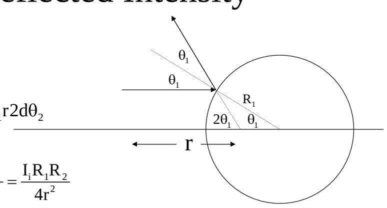
Large objects compared to the wavelength

Reflected Intensity

$$ds_1' = r2d\theta_1$$

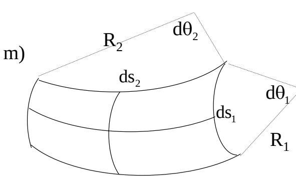
$$dA = ds_1' ds_2' = r2d\theta_1 r2d\theta_2$$

$$I_r = \frac{dP}{dA} = \frac{I_i R_1 d\theta_1 R_2 d\theta_2}{r2d\theta_1 r2d\theta_2} = \frac{I_i R_1 R_2}{4r^2}$$



$$TS = 10\log\left(\frac{I_r}{I_i}\right)$$ (At r = 1 m)

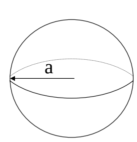
$$TS = 10\log\left(\frac{R_1 R_2}{4}\right)$$



Special Case – Large Sphere

$$R_1 = R_2 = a$$

$$TS = 10\log\left(\frac{a^2}{4}\right) = 20\log\left(\frac{a}{2}\right)$$



Note:

$$\frac{\sigma}{4\pi} = \frac{a^2}{4}$$

$$\sigma = \pi a^2$$

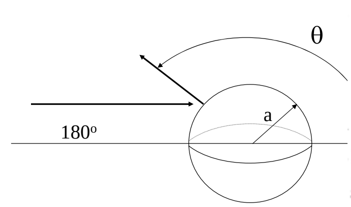
Large means circumference >> wavelength

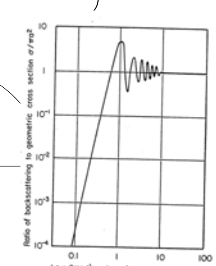
$$ka \gg 1$$

TS positive only if a > 2 yds

Large Spheres (continued)

$$\frac{I_r}{I_i} = \frac{1}{4\pi r^2}\left(\pi a^2 + \pi a^2 \cot^2\left(\frac{\theta}{2}\right) J_1^2\left(ka\sin\theta\right)\right)$$





Example

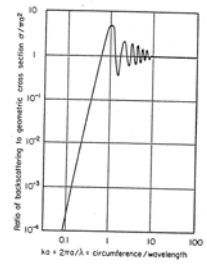
- An old Iraqi mine with a radius of 1.5 m is floating partially submerged in the Red Sea. Your minehunting sonar is a piston array and has a frequency of 15 kHz and a diameter of 5 m. 20 kW of electrical power are supplied to the transducer which has an efficiency of 40%. If the mine is 1000 yds in front of you, what is the signal level of the echo. Assume spherical spreading.

Scattering from Small Spheres (Rayleigh Scattering)

$$\frac{I_r}{I_i} = \frac{\pi^2 V^2}{\lambda^4 r^2}\left(\frac{3}{2}\cos\theta - 1\right)^2$$

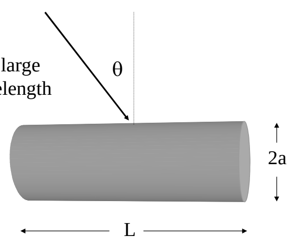
$$TS = 10\log\left[\frac{25}{36}(ka)^4 a^2\right]$$

$$ka < 1$$



Scattering from Cylinders

Dimensions (L,a) large compared to wavelength



$$TS = 10\log\left[\left(\frac{aL^2}{2\lambda}\right)\left(\frac{\sin\alpha}{\alpha}\right)^2 \frac{\cos^2\theta}{(1yd)^2}\right]$$

$$\alpha = \frac{2\pi L}{\lambda}\sin\theta$$

$$\boxed{\theta = 0^o} \Rightarrow TS = 10\log\left[\left(\frac{aL^2}{2\lambda}\right)\frac{1}{(1yd)^2}\right]$$

Gas Bubbles

$$\sigma_{bs} = \frac{a^3}{\left(\left(\frac{f_0}{f}\right)^2 - 1\right)^2 + \delta^2}$$

$f_0 \equiv$ resonant frequency

$\delta \equiv$ damping term

$$f_0 = \frac{1}{2\pi a}\sqrt{\frac{3\gamma P_w}{\rho_w}} \approx \frac{3.25}{a}\sqrt{1+0.1z}$$

$\rho_w = 1000$ kg/m^3

$P_w \equiv$ hydrostatic pressure in Pa $\left(\approx 10^5(1+0.1z)\right)$

$z \equiv$ depth in meters

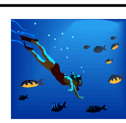
$\gamma \equiv$ adiabatic constant for air (≈ 1.4)

- Damping effect is due to the combined effects of radiation, shear viscosity and thermal conductivity. A good approximation is

$$\delta \approx 0.03 f_k^{0.3} \text{ for } 1\text{ kHz} < f_k < 100\text{ kHz}$$

- where f_k is the frequency in kHz.

Fish



- Main contribution for fish target strength comes from the swim bladder.
- This gas-filled bladder shows a very strong impedance contrast with the water and fish tissues. It behaves either as a resonator (frequencies of 500 Hz-2 kHz depending on fish size and depth) or as a geometric reflector (> 2 kHz). This swim bladder behaves very similar to gas bubbles. The difference in target strength between fish with and without swim bladder can be 10-15 dB.
- A semi-empirical model most often used is:

$$TS_{fish} = 19.1\log L + 0.9\log f_k - 24.9$$

- Love (1978)
- This formula is valid for dorsal echoes at wavelengths smaller than fish length L.

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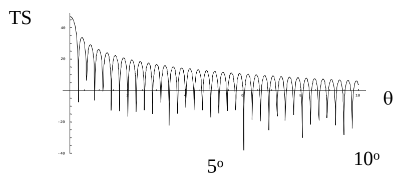
Infinitely long thin cylinder	$\frac{9\pi^4 a^4}{\lambda^2}r$	a = radius of cylinder	Normal to axis of cylinder	ka<<1
Finite cylinder	$\frac{aL^2}{2\lambda}$	L = length of cylinder a = radius of cylinder	Normal to axis of cylinder	ka>>1 r > L^2/λ
	$\frac{aL^2\left(\sin\beta/\beta\right)^2\cos^2\theta}{2\lambda}$	a = radius of cylinder β = kLsinθ	At angle θ with normal	
Infinite Plane surface	$\frac{r^2}{4}$		Normal to plane	
Rectangular Plate	$\left(\frac{ab}{\lambda}\right)^2\left(\frac{\sin\beta}{\beta}\right)^2\cos^2\theta$	a,b = sides of ractangle β = ka sinθ	At angle θ to normal in plane containing side a	r > a^2/λ kb >> 1 a > b
Ellipsoid	$\left(\frac{bc}{2a}\right)^2$	a, b, c = semimajor axis of ellipsoid	parallel to axis of a	ka, kb, kc >>1 r >> a, b, c
Circular Plate	$\left(\frac{\pi a^2}{\lambda}\right)\left(\frac{2J_1(\beta)}{\beta}\right)^2\cos^2\theta$	a = radius of plate β = 2kasinθ	At angle θ to normal	r > a^2/λ ka>>1

Example

- What is the target strength of a cylindrical submarine 10 m in diameter and 100 m in length when pinged on by a 1500 Hz sonar?



Example

- What is the target strength of a single fish 1m in length if the fish finder sonar has a frequency of 5000 Hz?